

# **The Right Price? Prices in a Dynamic Input-Output Model**

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# The Right Price? Prices in a Dynamic Input-Output Model

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**Abstract** *The fundamental input-output price identity captures the interrelationships between commodity prices across sectors and value added. Nominal output of each commodity is equal to the sum of intermediate input costs and value added. Historical estimates of real commodity output are most often derived using price deflators to deflate nominal output, or indexes of production, if available. If a balanced series of nominal input-output (IO) tables are compiled, constant price IO tables can be constructed that automatically ensure the satisfaction of the price identity.*

*In an extended dynamic IO model, both prices and quantities are calculated. Conditions of macroeconomic and industry tightness and slack affect prices, and prices affect consumption, investment, imports and exports. In certain models, prices also can affect intermediate demands. How these prices are calculated thus has important implications for the behavior of the model. An important question is the consistency of the prices calculated in the forecast with the historical deflators compiled to construct the constant price IO tables. Two examples we examine are hedonic deflators for computers and other goods, and the deflators for wholesale and retail trade.*

## 1 Background

The two fundamental input-output identities show the clear interrelationships between the production of different commodities, and the cascading effects of price changes through the economy. They also suggest an operational method for the calculation of outputs and prices in a multisectoral model. The price equation is often stated simply as:

$$p' = p'A + v' \tag{1}$$

where  $p'$  is a row vector of commodity prices, and  $v'$  is a row vector of unit value added (total value added divided by real output) by commodity, and  $A$  is the direct requirements matrix.

The concept is an old and valuable one in economics, tying factor incomes to the formation of prices, with causality flowing in both directions. Although roots go back to Walras and classical economists such as Ricardo, the price interrelationship was raised from inchoate ideas into methods of operational measurement in Leontief's 1928 "Economy as Circular Flow", the measurement being achieved and presented in several papers published in the 30s and 40s.<sup>1</sup> Leontief's message, to both economists and statisticians, was that wages, profits, taxes and prices were interdependent, both within a single industry, and across industries. He demonstrated how an increase in wage rates across industries would have differential impacts on prices by industry, using a static model with about 19 industries.

Leontief's quantity model was widely adopted as the "input-output model", and the price model has received less attention. Nevertheless, it is used in applied general equilibrium and econometric input-output models to estimate the price impacts of carbon taxes, energy price changes, changes in technology and multi-factor productivity, and labor productivity. In what has sometimes been called the extended input-output model, quantities affect prices, and prices affect quantities. In such a model, the overall level of economic activity, as measured by actual to potential GDP, the unemployment rate, or growth of real sectoral output may be

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<sup>1</sup> Leontief (1928) was a publication of ideas developed in his Ph.D. thesis, but still with no clear concept of an input-output table. Leontief (1937, 1946, 1947) show for the first time the use of the price relationships with U.S. data.

used as an explanatory variable for sectoral prices or profits. In turn, relative prices may be used in a consumer demand system, or as variables to explain imports, equipment investment or construction. This interrelationship of quantities and prices may go some way to addressing neoclassical and neo-Keynesian criticisms of input-output modeling.<sup>2</sup>

Once understood, the Leontief price identity seems incontrovertible, almost tautological. For consistent accounting, as well as consistent calculation of quantities and prices, the identity must hold. However, most statistical agencies compile price deflators without consideration of the implications of consistency of these deflators with intermediate cost and value added. Although constant price IO tables can still be calculated with these deflators, the results are sometimes strange, particularly with hedonic indexes that decline rapidly over time.

This paper explores several aspects of the IO price calculation. After reviewing methods of calculating commodity prices in a dynamic model in sections 2, we turn to the derivation of the constant price IO table in section 3. Section 4 explores the derivation of value added by commodity, if one has only industry data at hand. The relationship between IO commodity deflators and published purchasers' price deflators for personal consumption and investment is not satisfactorily resolved, at least with US data. Section 5 touches on this issue. Sections 6 and 7 deal with hedonic price indexes and wholesale and retail price deflators respectively, two areas where adherence to the price identity may need to be considered. Section 8 concludes, and offers some suggestions.

## 2 Calculation of Commodity Prices

In a closed economy, with a constant direct requirements matrix  $A$ , and only one price for each commodity, the calculation of the Leontief inverse and solution of equation (1) using:

$$p' = (I - A)^{-1}v' \quad (2)$$

is certainly feasible. But in a more general formulation, import prices need to be considered, as well as the possibility that the input-output coefficients may be responsive to relative prices or technical change. In any case, it is usually convenient to solve commodity prices using the Gauss-Seidel iterative technique.

In the simplest case:

$$p_j = \sum_{i=1}^n a_{ij} p_i + v_j \quad (3)$$

For the iterative Gauss-Seidel solution one first chooses a starting value for  $p$ , say  $p^0$ , which could be set equal to unity, to the value of prices in the previous year, or even to zero. The starting value does not matter for the final solution, but the choice of a good starting value can speed up the solution. Then write the value of  $p$  in the  $k$ th iteration as:

$$p_j^k = \sum_{i=1}^{i \setminus j} a_{ij} p_i^k + a_{jj} p_j^k + \sum_{i=j+1}^n a_{ij} p_i^{k-1} + v_j \quad (4)$$

which simplifies to:

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<sup>2</sup> Schumann (1990) contains an interesting characterization of suggested extensions. Bazzazan and Batey (2003) and Kratena (2005) provide different flavors of the extended model. The INFORUM family of models as characterized by Almon (1991) and Grassini (1997, 2001) certainly fit this description. The Cambridge model of Barker (1987) and his colleagues is quite similar in many respects.

$$p_j^k = \frac{\sum_{i=1}^{i \setminus j} a_{ij} p_i^k + \sum_{i=j+1}^n a_{ij} p_i^{k-1} + v_j}{1 - a_{jj}} \quad (5)$$

In each iteration, the value of  $p_j^k$  is compared to the value of  $p_j^{k-1}$  and the procedure continues until convergence within a predefined tolerance level.

To capture the effects of import prices, one may estimate an import matrix  $A^m$  and domestic requirements matrix  $A^d$ , such that  $A = A^d + A^m$ . Since presumably the import prices  $pm$  are not directly dependent on the domestic commodity prices  $pd$ , we can rewrite (5) as:

$$pd_j^k = \frac{\sum_{i=1}^{i \setminus j} pd_i^k a_{ij}^d + \sum_{i=j+1}^n pd_i^{k-1} a_{ij}^d + \sum_{i=1}^n pm_i a_{ij}^m + v_j}{(1 - a_{jj})} \quad (6)$$

This Gauss-Seidel iterative technique is preferable to a simple multiplication of unit value added by the Leontief inverse, for several reasons:

1. If A-matrix coefficients are specified as a function of prices, relative prices, or other variables, they can be calculated as the iterations proceed.
2. If individual prices or groups of prices need to be modified or fixed exogenously, this can be done as the iteration proceeds, in a way that maintains consistency.
3. More complicated price specifications can be handled, such as the case where there are two or more prices for a commodity across the row. An example would be the case of separate prices for exports, due either to a different product mix, or the pressure of foreign competition.

### 3 The Constant Price IO Table

In solving a dynamic input-output model either in historical simulation or in a forecast, it is necessary to express the table in constant prices, in order to make valid comparisons of calculations over time. In Leontief's original exposition<sup>3</sup> and later essays, he emphasized that the flows in the table should be proportional to quantities, such as yards of cloth, bushels of corn, or tons of steel. In practice, even at fine levels of detail, the commodities in the table are still aggregates of many real world commodities, so the quantities and prices are necessarily dollar values and price indexes. Several methods have been developed for deriving the constant price input-output table<sup>4</sup>, but many of these start from the proposition that the input-output table in constant prices must sum down the column to constant price output and therefore maintain the necessity for deflating value added.

It is logical and straightforward to define each coefficient as deflated input divided by deflated output. If this approach is taken, it is apparent that flows will not sum to output down the column in constant prices, unless one is willing to accept unreasonably large or small (even negative) amounts of real value added.

A similar problem to adding up values of disparate commodities in constant prices down the column arises in lesser form in adding up across the row. It is common to have different price

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<sup>3</sup> Leontief (1936).

<sup>4</sup> Dietzenbacher and Hoen (1998) demonstrate a biproportional projection method, which uses known margins in constant prices. Rampa (2008) uses reliability weights and a modified Stone-Champernowne weighted least squares technique to derive the constant price table.

deflators for imported commodities, and sometimes different price series are available for goods destined for export versus domestic consumption. Despite possible problems due to different mixes of output in domestic use, imports and exports, it still seems sensible to define the constant price sum across the row as a meaningful aggregate, which should sum up either in current or constant prices. This is because the goal of the deflator is to convert the nominal value into something that moves like quantities of the given commodity.

If one accepts this premise, a deflator needs to be derived to deflate the domestically produced and consumed part of intermediate and final demand.

In current prices, the identity is

$$pd'q = pd'A^d q + pm'A^m q + pd'of^d + pm'of^m - pm'm \quad (7)$$

In constant prices

$$q = A^d q + A^m q + of^d + of^m - m \quad (8)$$

where:

$of^d$  = other (non-imported) final demand satisfied by domestic production

$of^m$  = other final demand satisfied by imports

$A^d$  = domestic direct requirements matrix

$A^m$  = imported direct requirements matrix

Note that since

$$m = A^m q + of^m$$

The imported and domestic components of intermediate and final demand can be separately deflated, and there is no additional complication in computing the domestic price.

#### 4 Value Added by Commodity

There are several alternative methods available for deriving direct requirement tables from a supply and use table. Although there is not unanimous agreement, there are clear advantages to solving an input-output model with symmetric commodity-by-commodity tables.<sup>5</sup> Except for clear cut cases of joint production or by-products, the product technology clearly describes the column of industry coefficients observed in the use table as a weighted average of commodity technologies of the component products. However, even if industry technology is chosen, value added is logically converted to the commodity basis along with the elements of the IO table. Value added by commodity can then be thought of as the labor and capital income allocated to each commodity produced by an industry.<sup>6</sup>

In order to update the commodity-by-commodity framework, current time series of value added are needed. Time series of value added data are usually available only by industry. Forecasting prices in a dynamic input-output model may require forecasts of value added, and it may be preferable to disaggregate value added into the components available from the national accounts, such as wages and salaries, supplemental benefits, corporate profits,

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<sup>5</sup> Almon (1970, 2000) has described an iterative method that reliably obtains product-to-product IO tables using commodity technology with no negatives. Miller and Blair (2009), in chapter 5 review alternative methods for obtaining analytical IO tables, including the Almon method.

<sup>6</sup> Especially with regard to capital income, production of commodities may more often be joint, as production takes place in the same building, or using some of the same tools and machines.

proprietor income, capital consumption, and taxes on production and imports, among others. The estimation of these forecasting equations must logically rely on historical time-series of industry data on wages, corporate profits, etc.

Following this path, we are soon faced with the problem of converting the updated or forecasted value added by industry to value added by commodity, in order to use commodity value added to calculate commodity prices. One solution is to start with a make matrix, and modify it by RAS or other updating method to obtain a balanced matrix that shows the relationship between commodity and industry value added in a base year, or at least a year in which both vectors are available. This method assumes that industries produce value added by commodity similar to the structure of commodity output.

Specify  $t=0$  as the base year, which may be the year of a detailed benchmark IO table, or of an updated annual IO table. Let  $NC$  be the number of commodities in the IO table, and  $NV$  be the number of industries for which value added is available in the national accounts. Let  $V^0$  be the above described matrix consisting of estimated value added by industry by product in the base year, of dimension  $NV$  by  $NC$ . Following Almon (1983), we will call this matrix the *Product-Industry Bridge*. Let  $va^C$  be the vector of commodity value added, and  $va^I$  be the vector of industry value added.

In years  $t$  beyond the period of availability of the IO data, we may have available data on final demands by product, and value added by industry. The final demands can be used to perform an IO output solution, and derive commodity value added as the difference between output and the sum of intermediate inputs, in current prices. Form a matrix  $W^0$  by normalizing  $V^0$  to sum to unity down the column.

$$W^0 = V^0 \widehat{va_0^C}^{-1}$$

If we define value added allocated ( $vaa$ ) as

$$vaa_t = W^0 va_t^C$$

This is the value added by industry that would have obtained if the base year share of each product were made in each industry. This will most likely not be equal to the data on value added by industry in period  $t$ . In the year of available data on final demand and value added beyond the available IO data, a discrepancy vector  $d$  is calculated

$$d_t = vaa_t - va_t^I$$

As the model forecasts, econometric equations make forecasts of each category of value added. These are placed into a matrix  $G$ , which has  $NV$  rows, and enough columns to hold each category of industry value added, plus the discrepancy vector  $d$ . Construct a function of output called *revawo*, for Real Value-Added-Weighted Output. This is defined as

$$revawo_{it} = \sum_j V_{ij}^0 \frac{q_{jt}}{q_{j0}}$$

where

$q_{jt}$  denotes output of commodity  $j$  in period  $t$ , and  $t=0$  is the base year

In the base year, *revawo* will be equal to *vaa*. In forecast years, it will grow according to the weights in  $V^0$ , times the growth of the respective commodity outputs that are produced by each industry  $i$ . Note that this formulation does not assume that the product composition of an industry's output is constant. If the output of product 1 grows faster than the output of product 2, the weight of product 1 will increase relative to that of product 2 in all industries that produce both of them.

Total value added plus the discrepancy will be the row total of the  $G$  matrix. We will denote this vector as  $g$ . Then the  $V$  matrix can be projected with

$$V_{ij}^t = \frac{V_{ij}^0}{\text{revawoit}} \left( \frac{q_{jt}}{q_{j0}} \right) g_{it}$$

Unit value added  $v$  is then

$$v_{jt} = \frac{\sum_i V_{ij}^t}{q_{jt}}$$

This result is then used to solve for commodity prices using the price identity. Actually, the price identity can be used to solve for  $p$  given  $v$ , or for  $v$  given  $p$ . With some commodities such as crude oil, minerals or agricultural commodities, it may be easier and more realistic to model prices than to model value added. In this case, once all prices have been determined, new row totals can be derived for the  $G$  matrix, and the resulting difference from the originally calculated value added can then be allocated. A common solution is to adjust corporate profits, or some component of operating surplus. For example, a low price of oil would in this case result in low profits in the oil industry.

In the US and other countries, the national accounts value added by industry does not sum to GDP, but rather to GDI (gross domestic income), which differs from GDP by the statistical discrepancy. In the IO framework, total value added must be equal to GDP. How should this discrepancy be distributed?

At the level of total value added by industry, the U.S. Bureau of Economic Analysis has successfully used the Stone-Champernowne generalized least squares method to revise the distribution of value added on the GDI basis to a GDP basis, based on the incorporation of reliability weights by industry and by value added category. This method essentially allocates the statistical discrepancy to industry, and these researchers have decided to allocate the discrepancy to the components of gross operating surplus<sup>7</sup>. A supplemental worksheet is available that shows value added by industry on the GDP basis, by several detailed subcomponents.<sup>8</sup> Comparing value added by industry from this source with that published in the national accounts reveals how the statistical discrepancy was allocated to industries.

## 5 Bridge Matrices and Prices in the National Accounts

In several countries, consumption and investment bridges are available which relate consumption by category, or investment by purchasing industry to commodity final demand. The columns of these bridges sum to 1.0, so the bridge can be used to form weighted price deflators for consumption and investment. However, these often differ substantially from the consumption or investment deflators published by the statistical agency. What might be the sources of this inconsistency, and how should they be handled?

The U.S. Bureau of Economic Analysis makes a consumption bridge available with the Benchmark IO table, and also with the series of annual IO tables. The benchmark bridge shows detail for about 200 categories of personal consumption to slightly less than 400 commodities, including transportation and trade margins. The annual bridge tables include 85 categories of consumption that may be translated to up to 72 commodities. For many consumption categories, the bridge translates to a single commodity, plus the margins. For example, in the annual IO bridge, the consumption category “New motor vehicles” translates to the commodity “Motor vehicles, bodies and trailers, and parts”, plus margins for wholesale and retail trade, and transportation. On the other hand, the category “Furniture and furnishings” generates final demand for 14 out of 72 different commodities, plus the margins.

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<sup>7</sup> See the paper by Rassier, et al. (2007) for the description of the method, and a later paper by Rassier (2012) for an investigation of the sources of the discrepancy.

<sup>8</sup> This is part of the GDP by Industry release, which is comparable to the value added data in the annual industry accounts. See <http://www.bea.gov/industry/index.htm>.



The national accounts include a tables of time series of personal consumption by category, in both current and constant prices, so a price deflator exists for each consumption category. Using a few simple assumptions, one can derive a consumption price based on the domestic commodity producer prices, import prices, and the consumption bridge. Form the import share  $s$  as

$$s_i = m_i / (q_i + m_i)$$

where:

$m_t$  = imports of commodity  $i$

$q_t$  = output of commodity  $i$

A coefficient matrix can be formed from the consumption bridge by normalizing down the column. If we denote that consumption bridge as  $BR$ , then the imported and domestic part of the bridge can be calculated as

$$BR_t^m = BR \hat{s}_t \quad \text{and} \quad BR_t^D = BR - BR_t^m$$

If  $pc_j$  is the price of the  $j$ 'th consumption category, then that may be formed from the bridge coefficient matrices as

$$pc'_t = pm'_t BR_t^m + pd'_t BR_t^D$$

where

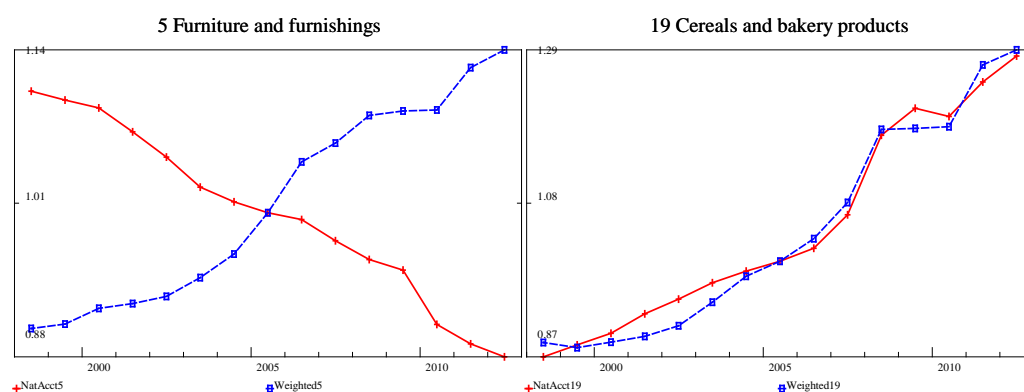
$pm'_t$  is the imported commodity price

$pd'_t$  is the domestic commodity price

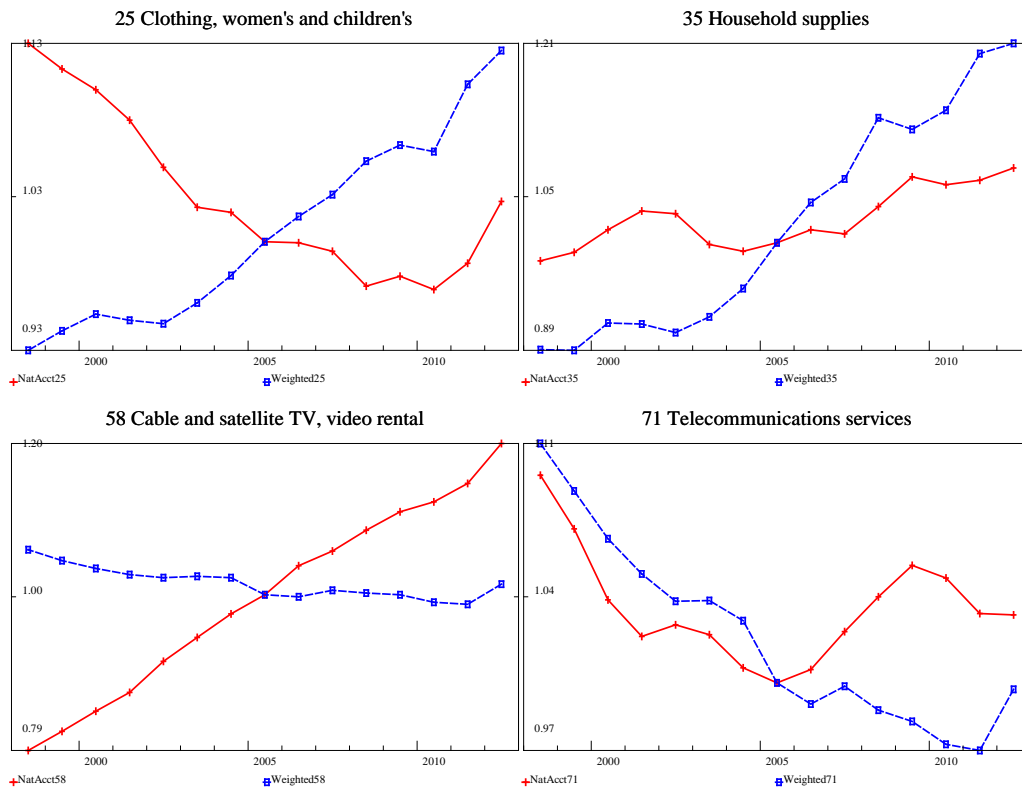
The intuition behind this simple identity is that the consumption price should be a weighted combination of the domestic and import prices of the commodities that make up that consumption category, including the prices of the trade and transport margins.<sup>9</sup>

It is perhaps instructive to compare the price constructed in this way with the deflators published in the national accounts tables. Figure 1 contains comparison charts for 6 selected consumption categories.

**Figure 1. Consumption Prices in National Accounts (NatAcct) Compared with Those Derived from IO Prices and Consumption Bridge (Weighted)**



<sup>9</sup> Prices of trade margins are discussed further in section 7 below.



These charts are not representative, as nearly 50 of the 85 categories do correspond fairly closely. However, they do show that for several common items, such as clothing and furniture, the prices derived from the consumption bridge and commodity deflators can be much different than the official national accounts deflator for that item. For Clothing and Furniture and furnishings, the source of the discrepancy may have to do with mismeasurement of import prices, as these are commodities with a high import share. In other cases, such as Cable and satellite TV or Telecommunications services, either the definition of the price concept and/or the data source may be different.

These differences pose problems both for the construction of the historical data as well as for forecasting:

1. For the compilation of constant price historical consumption data, one needs to decide which deflator to use, the official national accounts deflator, or one consistent with the IO commodity prices and the bridge. There are benefits to adhering to the published prices, but also benefits from internal consistency.
2. For forecasting, it is preferable to start with the forecast of commodity prices, and then build up the consumption deflators using the bridge. If the official deflators are used in the historical data, this implies a linking problem in the first year of the forecast.
3. If one starts by forecasting consumption by category in constant prices in a consumer demand system, and then passing this result through the bridge, the constant price totals will be equal. Only if the IO-derived prices are used to reflate consumption by category to current prices will the nominal totals of consumption by commodity and consumption by consumption category be equal. This is necessary to preserve total GDP in current prices.

A similar problem arises for equipment investment in the use of a capital flow table or investment bridge. Space does not permit a more detailed presentation, but we should note that additional problems arise due to the use of hedonic deflators for many categories of

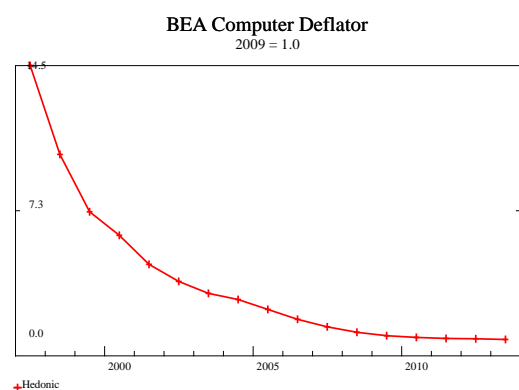
equipment, particularly computers and communications equipment, but also many others. It is quite common to find hedonic indexes for the total investment by industry, but no hedonic deflators for the commodities making up that investment. In other cases, both sets of data are deflated with hedonic indexes, but the indexes are different. The next section addresses the more general topic of hedonic prices and their incorporation into constant price IO tables and dynamic IO models.

## 6 The IO Implications of Hedonic Indexes

Consumer and producer prices for some commodities are constructed as hedonic indexes, which may decline rapidly, especially for computers, semiconductors, and other electronic products such as communication equipment<sup>10</sup>. While there are alternative methods for the calculation of hedonic price indexes, the core idea is that the quantity or constant price value of a good or service can be measured in terms of its desirable characteristics, and the job of hedonic estimation is putting a price on those characteristics, either using regression analysis or a matching model.<sup>11</sup>

Zvi Griliches, whose name is closely associated with the development of hedonic indexes, noted that most of the literature has focused on the demand or utility side, with relatively less attention to the cost side<sup>12</sup>. In a general equilibrium model with IO price linkages, the cost relationships between industries need to be considered. If hedonic indexes are adopted, the implications for the cost structure should be spelled out.

**Figure 2. BEA Computer Deflator**



We will focus on the BEA computer (NAICS 334111) deflator, this version from the Gross Output series.<sup>13</sup> As shown in Figure 2, the published price deflator declines from 14.5 to 0.8 from 1997 to 2013. This is a reduction in price by a factor of 18, over a period of 16 years. In other words, this implies that a given dollar value of expenditure on a computer in 2013 buys 18 times as much ‘real’ computer as in 1997.

To compile a time-series of constant price tables, the coefficients are constructed as deflated input divided by deflated output. Some of the larger inputs into computers (computer storage devices, computer peripherals, semiconductors, printed circuit board assemblies) are also hedonically deflated, and have deflators that are declining over time. However, as can be

<sup>10</sup> In the US, there is now a long list of products and services with price indexes constructed using hedonics, including automobiles, aircraft, buildings, and some medical services.

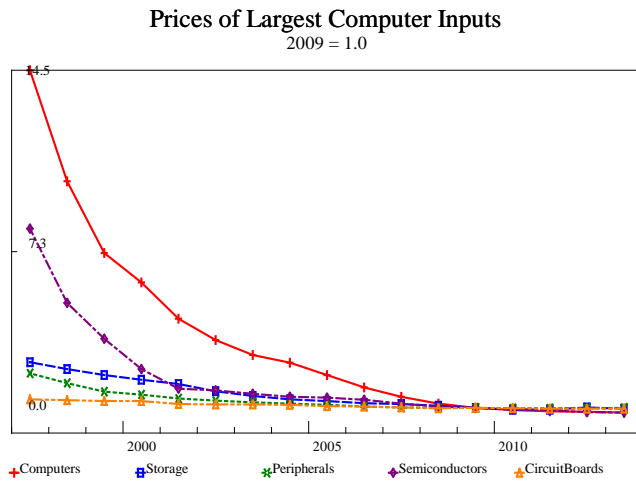
<sup>11</sup> Hulten (2003) provides a good review of hedonic price indexes, their history, and some theoretical issues.

<sup>12</sup> Griliches (1991).

<sup>13</sup> Published currently (2015) at [http://www.bea.gov/industry/xls/io-annual/GDPbyInd\\_GO\\_NAICS\\_1997-2013.xlsx](http://www.bea.gov/industry/xls/io-annual/GDPbyInd_GO_NAICS_1997-2013.xlsx).

observed in Figure 3, these prices are not declining as fast as the computer deflator. Almost all of the other inputs besides these have rising prices. This implies that the average IO coefficients in the computer column must fall monotonically, if we build the constant price IO tables in the standard way. The falling output deflator implies that real output will grow more rapidly than nominal output. If value added grows at about the same rate as nominal output, then unit value added (value added divided by real output) will tend to decline as well.

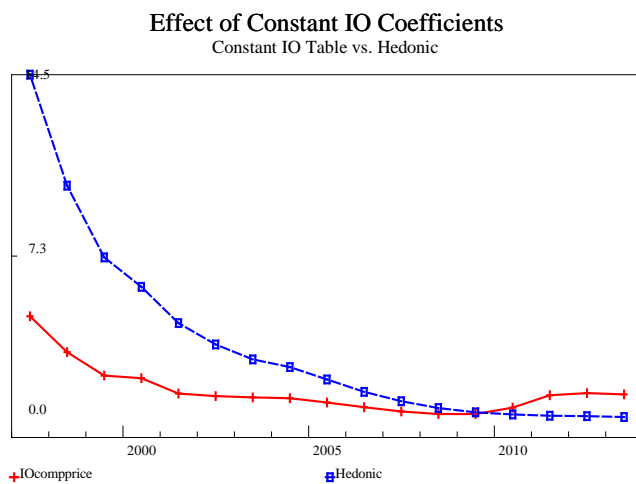
**Figure 3.**



The resulting constant price IO tables will still be consistent with the falling hedonic deflators, and the price identity will still hold in the historical data. Keep in mind that if the deflator is expected to continue to fall in the future, the forecasted IO coefficients in the computer column must continue their downward trajectory.

It may be interesting to quantify the extent of coefficient change necessary to achieve consistency with the hedonic deflator. If the IO coefficients are kept constant, and the price identity (2) is used to solve for the computer deflator, one obtains the red line ('+'s) in Figure 4.<sup>14</sup>

**Figure 4.**



<sup>14</sup> To construct unit value added  $v$ , real output  $q$  was constructed using the BEA deflator. If one believed that the computer deflator should fall less rapidly, this imparts downward bias to the red line. (A more rapidly declining deflator causes real output to grow more quickly, which causes unit value added to rise less quickly.)

The hedonic deflator is the blue line (squares). In other words, if one did not have a revised IO table for every year, and used a benchmark table to compute prices, this is what would be obtained for the computer price.

Note that the falling computer deflator also implies a declining level of nominal wages to real output, and rapidly increasing average labor productivity. In forecasting both of these variables, one must rely heavily on the projection of the computer deflator.

## 7 Prices of Wholesale and Retail Trade

The prices of wholesale and retail trade play a large role in the IO price calculation, and also in the calculation of prices involving the consumption and investment bridges. In personal consumption, these trade margin commodities make up a large share of the final purchaser's price of consumption by category. How should the prices for the wholesale and retail trade commodities be defined? This question is closely related to the definition of the measure of real wholesale and retail trade output.

Nominal output of the trade margin industries in the US IO data is defined by the Bureau of Economic Analysis as gross margin (sales less the cost of goods sold) plus commodity taxes, inventory valuation adjustment, misreporting adjustments, own-account production, and other adjustments<sup>15</sup>. The concept of retail trade output includes services that are valued by consumers in addition to the merchandise itself, such as convenient hours and locations, a selection of different types of merchandise, speedy or efficient check-out, and information about products. Some of these services may be produced with the staff of sales clerks, cashiers and warehouse and stock clerks. Wholesale trade output includes other services valued by producers or other wholesalers. All of these services cost resources to produce, such as intermediate inputs, labor and capital.

In the published US benchmark use table, wholesale and retail margins may be associated with each transaction. Retail margins are concentrated in the consumption bridge, but some business purchases of intermediate inputs or even capital goods are made from retail establishments. When the use table is simplified to a table in producers' prices, the wholesale and retail margins of on all intermediate inputs are combined into the wholesale and retail transaction for the purchasing industry, and so may be combined with non-margin output.<sup>16</sup>

Table 1 shows the distribution of trade output by wholesale trade and four retail trade commodities, in the 2007 benchmark IO table, along with the percentage shares by major destination<sup>17</sup>. Wholesale trade margins are dominant in the intermediate sector, but also important for personal consumption, investment and exports. The largest share of the retail trade margins are in consumption, but Other retail has a significant amount of sales to intermediate.

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<sup>15</sup> Yuskavage (2006) contains a good discussion of current and alternative methodologies for the derivation of real retail trade output and margin prices.

<sup>16</sup> Non-margin trade output includes sales on consignment, where the goods are not actually owned by the trade establishment, and broker's commissions and payments for other services.

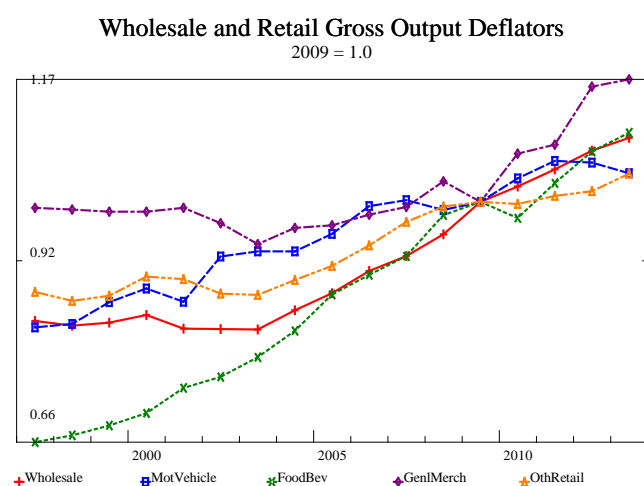
<sup>17</sup> Although Yuskavage (op.cit.) indicates that the unpublished worksheets for the table include over 50 categories of retail trade, the 2007 table is the first for which more than one retail trade commodity was included.

**Table 1.**

NAICS	Description	Intermediate	Personal				Government	Commodity Output
			Consumption	Investment	Exports	Imports		
<i>Trade Margin</i>								
420000	Wholesale trade	542,438	393,725	137,939	125,573	28,773	15,502	1,243,950
441000	Motor vehicle and parts dealers	10,827	119,068	12,213	-	-	-	142,108
445000	Food and beverage stores	1,336	161,338	587	-	-	-	163,261
452000	General merchandise stores	4,701	174,799	3,104	-	-	-	182,604
4A0000	Other retail	103,616	583,929	31,240	-	-	-	718,785
<i>Percent Shares by Destination</i>								
	Wholesale trade	43.6	31.7	11.1	10.1	2.3	1.2	
	Motor vehicle and parts dealers	7.6	83.8	8.6	-	-	-	
	Food and beverage stores	0.8	98.8	0.4	-	-	-	
	General merchandise stores	2.6	95.7	1.7	-	-	-	
	Other retail	14.4	81.2	4.3	-	-	-	

The BEA Gross Output data is on the same classification as the benchmark table, and so includes a time series of gross output and deflators for wholesale trade and these 4 categories of retail trade. Figure 5 compares these deflators over the period 1997 to 2013.

**Figure 5**



The BEA deflators shown above are constructed as weighted averages of sales deflators, which have been compiled from BLS producer price indexes and BEA purchasers' price indexes of personal consumption by detailed merchandise line. The weights are constructed based on Census data which shows the mix of goods sold by each type of retail establishment. In other words, if a retail industry that sold mostly computers were shown, that retail trade deflator would be declining, like the hedonic index for computers. Although all of these deflators rise after 2005, the deflators for Wholesale, Other retail, and General merchandise stores are all nearly flat from 1997 to 2005.

Two aspects of the constant price IO table in producers' prices are notable. First, in converting to constant prices, the IO coefficients in the trade industry columns will be adjusted for the relative price changes of the trade inputs and the trade output deflators. Forecasts or projections of the trade deflators will be based on input-cost and the projection of trade industry value added. Second, since the trade rows of this IO table are a combination of the trade margins applied to all inputs in the column, the trade IO coefficient is the deflated sum of these margins divided by deflated output. The same price is used to deflate each trade margin cell across the row. Forecasts of the prices of other industries will depend partly on the forecast of the trade prices. Forecasts of the purchasers' prices in the consumption and investment bridges will also be dependent on the forecast of the trade margin prices, with the

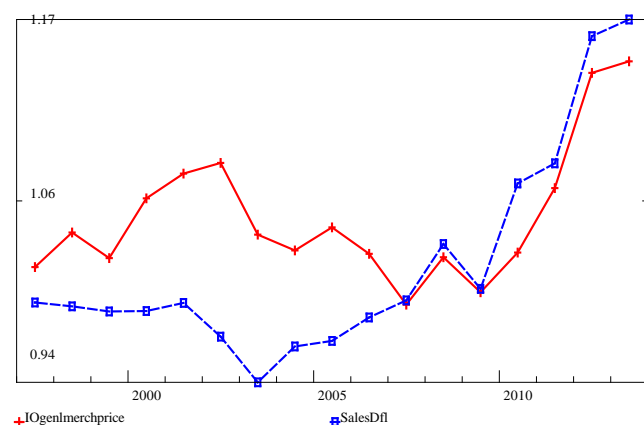
same price deflator used in all columns of the intermediate coefficient matrix and in the bridges.

Ideally, the historical price estimate constructed using retail sales deflators would be consistent with the input cost and value added Leontief price identity (2). Intuitively, this means that the price of retail sales should reflect the cost of production of retail sales output. How this price compares to the weighted average of the prices of the goods being sold remains to be determined.

I have made a rough comparison of the consistency in Figure 6 for the retail sector General merchandise from the US benchmark IO table. The blue line (marked with plus symbols) is the historical retail price based on sales deflators. The red line ('+'s) is calculated using identity (2) with a constant IO column, historical input deflators, and unit value added.

**Figure 6**

**Genl Merchandise: Effect of Constant IO Coefficients**  
Constant IO Table vs. Sales Deflators



In this case, the deflators are similar after 2007, but diverge somewhat over the 1997 to 2006 period.

The construction of trade margin deflators using weighted averages of the commodities sold by each type of wholesale and retail establishment assumes that the trade margin price should rise or fall according to prices of these commodities. If the intermediate, labor and capital cost per unit of margin also grow like these prices, then the sales deflator method will give a result consistent with the cost-based method. The graph above shows rough consistency, which is satisfying. However, if one were looking at more disaggregated types of retail, and included electronics stores, the margin price would fall at the average rate of the electronics goods sold, and probably not represent the change in the average cost of production. The sales deflator method ensures that purchasers' prices of such commodities move like their producers' prices, and the price of the trade margins will follow the producers' prices.

## 8 Conclusion and Suggestions

This paper has reviewed several aspects of the calculation and use of prices in the IO framework. The price relationship was central to Leontief's original vision, and is one of the two central identities formulated in his early work. It should be important to academic and applied economists, as well as to policy makers. The reasons why this relationship has received less attention than the quantity or output relationship may be due to difficulties in obtaining detailed price data, and to the fact that few statistical agencies publish the entire IO framework in constant as well as current prices. However, the use of the IO framework as a

tool for determining price effects is surely as useful as for determining impacts on production and employment. There have been many papers tracing the effects of changes in primary energy prices on the prices of other sectors, or of the effect of a carbon tax or similar energy taxes. The price relationship also leads to a consistent determination of the effects of commodity price changes on aggregate price deflators, such as the GDP deflator and the implicit deflator for disposable income, which has strong effects on personal consumption in a dynamic IO model.

The IO price relationships also hold great interest for statistical agencies. In many countries, these agencies have found that the compilation of IO data is not only valuable to academics, modelers and business, but also useful for the development and verification of the national accounts. The IO framework makes explicit the dependencies and interrelationships between the product side and the income side of the accounts in current prices, and help ensure greater reliability and consistency.<sup>18</sup>

The IO framework can potentially assist in the development of the constant price national accounts as well. For example, with an estimate of the import share by commodity, the implicit relationship between import shares and import prices and the deflators of final demand categories can be made explicit. In the US, the categories of consumption, equipment investment and construction shown in the expenditure tables are deflated without reference to a constant price IO framework. Section 5 illustrated briefly some inconsistencies in the deflators for several categories of personal consumption, we have found similar puzzles in the equipment and construction deflators. Resolution of these discrepancies could lead to better deflation of the product side of the national accounts. Finally, IO analysis focuses on the relationship between the intermediate costs and value added components with the product price. I have argued that the development of hedonic deflators and deflators for wholesale and retail trade may be improved by considering this relationship.

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<sup>18</sup> Stone (1961) gives several useful examples.



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