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On the Depreciation Sector of Jidea 6
- Trial Application of Various Methods -

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1. Introduction

The depreciation sector in the value added side of I-O table, though important as an integral part of the table, cannot be a center piece of I-O analysis. One of the reasons of this unhappy situation of the depreciation sector in the I-O analysis is that the capital stock which is deeply related to the depreciation is not playing important roles in the table. I-O system goes well without the detailed description of capital stock, though the depreciation sector cannot be ignored.

The simplest way to determine the level of depreciation within the framework of I-O table, will be to estimate the sectoral depreciation equation related to the investment of the same sector accumulated for last few years[1], or simply to fix the depreciation rate by sector, although how to choose the suitable level of depreciation rate is another problem. One of the laborious tasks inevitable for the study of depreciation sector in the I-O table is to prepare the capital matrix table to convert the investment by industry in final demand side to and from the sectoral investment by purchasers' side corresponding to the sectoral depreciation in the value added side. Fortunately for our Jidea 6 model, four capital matrices for 1985, 1990, 1995 and 2000 are available as explained in Sasai (2006), though rearranging the tables suitable for our model was another big problem.

The purpose of this study is, starting from the definitional equation of capital stock, to determine the sectoral depreciation rate and to find out the way to calculate the initial value of sectoral capital stock and eventually to obtain the sectoral capital stock. The method to calculate sectoral capital stock has already been introduced by

[1] This is what we have applied to the depreciation sector up to Jidea 5.

Professor Almon (1999)[2]. Our study was intended to be a small challenge to "the Bucket system", though our preliminary result was complete failure.

In the next section the method to determine depreciation equations and the initial value of capital stock will be presented, and the third section explains data and estimation results of the depreciation sector. In the fourth section other method to estimate depreciation equations will be discussed, and the final section is mainly for concluding remarks and suggestions for further study.

2. Method

As everyone knows, the capital stock at time t is the sum of the capital stock at time $t - 1$ and the net investment at time t. The following equation (1) is the definitional equation of capital stock in relation to the gross investment and the depreciation.

$$K_t = K_{t-1} + I_t - Dep_t \qquad (1)$$

K_t : Capital stock at time t,

K_{t-1}: Capital stock at time $t - 1$,

I_t : Gross investment at time t,

Dep_t : Depreciation at time t.

Equation (1) can be rewritten to the following equation (2).

$$K_t = K_0 + \Sigma(I_t - Dep_t) \qquad (2)$$

K_0 : Initial value of capital stock,

$\Sigma(I_t - Dep_t)$: cumulative sum of net investment.

Depreciation can be related to capital stock as expressed in equation (3) following,

$$Dep_t = \beta K_t = \beta \{K_0 + \Sigma(I_t - Dep_t)\} = \beta K_0 + \beta \Sigma(I_t - Dep_t) \qquad (3)$$

The depreciation equation will be estimated in the form of equation (4) below, a linear combination of Dep_t with $\Sigma(I_t - Dep_t)$.

$$Dep_t = \alpha + \beta \Sigma(I_t - Dep_t) \qquad (4)$$

Here, α stands for the constant term which should be positive, and β for the depreciation rate, which should be also positive and less than 1. One of the advantages of this specification (4) is that the depreciation equation can be estimated without the data of capital stock. Comparing equation (3) with equation (4), it is quite clear that βK_0 in equation (3) equals to the constant term α in equation (4), and that K_0, the initial value of capital stock, is obtained by dividing α by β.

Then the value of capital stock is calculated by adding this initial value of capital

[2] Basic formula of his bucket system is available in p.62. Its application was prepared by Hasegawa (2006) to create Japanese capital stock data. He renamed it as "Cascaded Leaky Bucket" system.

stock to the cumulative sum of net investment as shown in equation (2). The other much simpler way to obtain K_t is by dividing Dep_t by β.

Alternative forms of depreciation equation like the following (5) and (6) are often found as a simplified version of the equation.

$$Dep_t = \alpha + \beta \sum_{t-1}^{t-5} netI \qquad (5)$$

or,

$$Dep_t = \alpha + \beta \sum_{t-1}^{t-3} netI \qquad (6)$$

Here, $netI_t$ stands for net investment (I_t - Dep_t). In equation (5) and (6) the service life of the investment goods is assumed to be for five years and three years respectively. It may not be realistic to assume that the investment of all the sectors have same length of service life, if the main concern of the study is to determine the sectoral depreciation rates.

3. Data and the Result of Estimation

It is rather waste of time to repeat the data availability for the analysis of depreciation sector of Jidea 6, since it has already been explained by Ono (2006). Time series data of sectoral depreciation at nominal terms (dep%1), sectoral investment by purchasers' side at constant terms (invr%1) and at nominal terms (inv%1) are main players in this section. It may be also unnecessary to describe capital matrix table to convert the sectoral investment of suppliers' side (ipr%1 and iprr%1) to and from the sectoral investment of purchasers' side (inv%1 and invr%1).

First, sectoral depreciation data should be deflated to the series at constant terms of 2000 (depr%1) by the sectoral investment deflator. Then the time series of net investment (invr%1 - depr%1) were calculated. As already mentioned in section 2 above, cumulative sum of net investment is an explanatory variable in the depreciation equation. Results of estimation were summarized in table-1. Out of sixty six industrial sectors of Jidea 6 model, the depreciation rate of twenty five sectors can be determined by the equations type a and type b explained by cumulative sum of net investment lagged one year. They are sectors 3, 5, 6, 7, 9, 13, 15, 20, 21, 24, 25, 28, 32, 33, 40, 46, 48, 50, 55, 57, 60, 62, 64, 65 and 66. The first trial to estimate the depreciation equation is not so promising that the equations for sectors 2, 4, 10, 11, 14, 16, 17, 23, 30, 31, 34, 38, 44, 45 and 53 were estimated by type d including time trend as an additional variable to improve RBSQ of the equation. The second trial, based on a more simplified version of equation explained by the sum of gross investment for last three years or five years, were attempted. They are type f and type g. The results are also available in table-1.

Sectors 8, 18 and 59 can be included in the group of successful result. If we lower the level of RBSQ, one of the criteria to select the estimated equations, the number goes up to fifty five sectors including sectors in the shaded cells in table-1. We have three blank sectors of 43, 52 and 61. For the remaining eight sectors it is most appropriate to assume fixed rate of depreciation calculated as a ratio of depreciation in the terminal year of the observation period relative to the sum of investment for the last three or five years. The sum of investment for last few years works as a proxy of capital stock.

However, equations including time trend or the equations with simplified specification are not suitable for estimating initial value of sectoral capital stock. Thirty three sectors successful in the first trial, some of them are with low level of RBSQ though, could be selected to calculate the initial value of capital stock and the stock level of the sector.

The trial to calculate sectoral capital stock was withdrawn, because during the course of experiment it was informed that the latest estimation of private capital stock by eighty one sectors prepared by Shishido (2006) was available[3]. His data is expressed in real terms of 2000 and data availability is from 1970 to 2003. Shishido's estimate is one of the most reliable databases of the sectoral private capital stock in Japan. Starting from the detailed analysis on the *Census of National Wealth* of 1970, and keeping consistency with the government estimates of private capital stock of twenty six sectors, and with the help of depreciation rate available in the *Census of Manufactures* of Japan, disaggregation of the government estimates of twenty six sectors into time series of private fixed capital stock of eighty one sectors can be produced. This seems to be the orthodox and the most reliable way to estimate private sectoral capital stock of which availability should be much more publicized. In the next section alternative method of estimation of sectoral depreciation will be presented.

4. Alternative Method

The miserable result presented above is not without reason. The rate of depreciation by sector estimated in section 3 is a weighted average of depreciation rate of investment goods purchased by the sector. Then the alternative and seemingly more suitable method to estimate depreciation sector in the I-O model may be the following[4]. First, by means of capital matrix (table for 2000), sectoral depreciation (dep%1) should be converted to the depreciation corresponding to the sectoral investment of selling side

[3] Data is not yet open to public, but our team has access to the data through the courtesy of Dr Shishido. Compiling method is explained in Shishido (2006).

[4] Perhaps this method is already introduced elsewhere, though I could not locate the paper.

(ipr%1) in the final demand sector. This is named as fdep%1. With the help of investment deflator (ipr%1/iprr%1) fdep%1 in real terms of 2000 (fdepr%1) will be produced. Then the net investment in final demand side (iprr%1 - fdepr%1) can be defined. Same as the equation (4) in section 2, the cumulative sum of this net investment will be an explanatory variable in the sectoral depreciation equation with three dummy variables.

Finally, for the simulation purposes, the sectoral depreciation in final demand side (fdep%1) should be converted to the sectoral depreciation in value added side (dep%1) by means of capital matrix.

The sectoral investment of selling side (iprr%1) and the corresponding sectoral depreciation (fdepr%1) in the final demand of the table has thirty one sectors. Our expectation to have better result was not fulfilled. Out of thirty four sectors only seven sectors could be selected with good result. One of the big problems is net investment (netiprr%1) of some sectors in the final demand side thus calculated have always negative value suggesting overestimation of the converted depreciation (fdepr%1).

As a supplementary, final trial was to estimate equations of depreciation rate defined as the ratio of sectoral depreciation in real terms (fdepr%1) divided by the last three years or five years sum of gross investment in real terms (iprr%1). The equation is explained by time trend and three dummy variables. The dependent variable is in the logarithmic form, and growth rate of the depreciation rate during the observation period can be shown by the parameter of the time trend. In table-2 sectors marked with type w and type x were estimated by this type of equation. This formula was also adopted to estimate the equation of depreciation rate in the depreciation sector of value added side.

5. Conclusion

It is most appropriate to list up what has been done in this study and what should be done as a further analysis. Out of sixty six industrial sectors of Jidea 6 model the depreciation rate of twenty five sectors can be determined by the methods of estimation explained above, and if we lower the level of RBSQ, one of the criteria to select the estimated equations, the number goes up to fifty five sectors.

The method to determine the depreciation rate of sectoral capital stock presented in section 2 is theoretically correct but can not be interpreted as the depreciation rate in the real business world, since the depreciation rate of each category of capital investment goods is strictly determined in advance by account law or government tax law.

What is intended to in this analysis is, in the process estimating the sectoral depreciation equations in the value added side of I-O table of Jidea 6, to find out what will be the rate of depreciation in each sector as a weighted average of depreciation rate of investment goods purchased by the sector, though estimated results of some sectors were found out to have unrealistically high, for example, sector 9 (clothing) estimated by type a showing over eighty percent of depreciation rate which should be cross-checked by other sources of database of capital stock and depreciation.

To improve the poor estimation result of sectoral depreciation, alternative method was presented. It is to estimate the depreciation equation, using the depreciation data converted to the final demand side, in relation to the investment in the final demand side. Though the result was again complete failure, it is worthwhile to re-examine the data, and the capital matrix to convert the data in value added side to and from the data in the final demand side.

Effort, to produce the initial value of capital stock of each sector and the value of sectoral capital stock as a second product of this method of estimation, was withdrawn, because one of the reliable databases of sectoral capital stock of Japan was estimated and introduced by Shishido (2006).

One of the big problems is to solve the complexity in our capital matrix to convert the sectoral investment by sellers' side to and from the sectoral investment by purchasers' side. Ours is not square. It is a time consuming work to prepare capital matrix choosing sector number in one classification corresponding to the sector number in the other classification. Estimation of depreciation equations based on the depreciation data converted to final demand side should be re-examined, though the first trial was not successful.

References

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Table-1 Depreciation Sector (in the Value Added Side)

Preliminary Results of Estimation

Sector	Type	a	b	K0=a/b	RBSQ
1	f	1372	0.1083	–	0.472
2	d	44.2	0.1162	380.4	0.786
3	b	229.4	0.1509	1520.2	0.801
4	d	205	0.1357	1510.7	0.933
5	b	10.6	0.0478	221.8	0.662
6	b	750.0	0.0306	24509.8	0.911
7	b	289.8	0.0211	13734.6	0.957
8	f	78.0	0.1384	–	0.733
9	a	287.3	0.8323	345.2	0.693
10	d	636.8	0.1552	4103.1	0.775
11	d	726.4	0.2068	3512.6	0.777
12	b	619.2	0.0344	18000.0	0.454
13	b	388.6	0.0205	18956.1	0.876
14	d	1821.4	0.1095	16633.8	0.882
15	b	69.5	0.0180	3861.1	0.633
16	d	790.4	0.0058	136275.9	0.653
17	d	1887.7	0.1403	13454.7	0.967
18	g	16.1	0.1669	–	0.610
19	v	1.2322	−0.0316	–	0.775
20	b	477.5	0.1200	3979.2	0.909
21	b	165.5	0.0510	3245.1	0.751
22	b	138.5	0.1159	1195.0	0.521
23	d	1678.1	0.0554	30290.6	0.756
24	b	238.7	0.1799	1326.8	0.794
25	b	155.1	0.0339	4575.2	0.718
26	b	292.6	0.0958	3054.3	0.514
27	b	42.3	0.1568	269.8	0.464
28	b	150.4	0.0643	2339.0	0.778
29	b	1000.8	0.0318	31471.7	0.427
30	d	347.5	0.052	6682.7	0.773
31	d	1306.1	0.1433	9114.4	0.737
32	b	435.2	0.1228	3544.0	0.937
33	b	430.7	0.1422	3028.8	0.844
34	d	9302.9	0.3722	14191.6	0.836
35	d	5282.1	0.2702	3978.9	0.501
36	d	1075.1	0.0508	21163.4	0.450
37	b	97.3	0.1017	956.7	0.411
38	d	3827	0.0514	74455.3	0.649
39	v	10.3334	−0.1321	–	0.901
40	b	176.2	0.0110	16018.2	0.615
41	v	6.0858	−0.0858	–	0.839
42	a	346.8	0.0368	9423.9	0.424
43	–	Dependent variable is constant.(zero)			
44	d	5493.3	0.1349	40721.3	0.745
45	b	325.7	0.0787	4138.5	0.545
46	b	199.0	0.0108	18425.9	0.651
47	v	−7.6628	0.0667	–	0.862
48	b	161.1	0.0428	3764.0	0.596
49	f	192.6	0.0132	–	0.532
50	b	215.9	0.0501	4309.4	0.820
51	v	−1.1487	−0.0272	–	0.461
52	–	Dependent variable is constant.(zero)			
53	d	13661.9	0.2344	58284.6	0.743
54	g	3080.0	0.0092	–	0.428
55	b	293.4	0.0665	4412.0	0.942
56	u	−0.9551	0.0203	–	0.752
57	b	3632.4	0.0474	76632.9	0.907
58	v	−8.2402	0.0781	–	0.993
59	f	3150.8	0.0112	–	0.844
60	b	2057.3	0.0854	24090.2	0.964
61	–	Dependent variable is constant.(zero)			
62	b	4360.2	0.0444	98202.7	0.668
63	u	−1.464	0.008	–	0.272
64	b	3106.2	0.1035	30011.6	0.952
65	b	2332.1	0.0482	48383.8	0.918
66	b	874.4	0.0922	9483.7	0.632

Type of Equation

type a 1990–2004
depr = f(D2, D3, netkstk[1])

type b 1986–2004
depr = f(D1, D2, D3, netkstk[1])

type d 1986–2004
depr = f(D1, D2, D3, timet, netkstk[1])

type f 1988–2004
depr = f(D1, D2, D3, sum(invr[1] + ··· + invr[3])

type g 1990–2004
depr = f(D2, D3, sum(invr[1] + ··· + invr[5])

type u 1988–2004
rdepr =depr/sum(invr[1] + ··· + invr[3])
log(rdepr) = f(D1, D2, D3, timet)

type v 1990–2004
rdepr =depr/sum(invr[1] + ··· + invr[5])
log(rdepr) = f(D2, D3, timet)

Definition of Variables

depr: Depreciation in real terms of 2000

netkstk: Cumlative sum of net investment in real terms of 2000

invr: Gross investment in real terms of 2000

D1: Dummy variable; 1 for 1986, 87, 88, 89

D2: Dummy variable; 1 for 1991, 92, 93, 94

D3: Dummy variable; 1 for 1996, 97, 98, 99

timet: time trend

Estimation criteria

Except for type u and type v

Parameter a > 0

Parameter b posstive and less than 1

RBSQ > 0.6 (to be changed)

Table-2 Depreciation Sector in the Final Demand Side
Preliminary Results of Estimation

Sector	Type	a	b	K0=a/b	RBSQ
1	w	-9.1008	0.0914	-	0.815
8	w	-10.7852	0.1176	-	0.736
9	w	-5.7095	0.0728	-	-0.181
10	w	-8.3788	0.0973	-	-0.031
11	w	-11.9747	0.1195	-	0.729
29	-	Dependent variable is constant.(zero)			
30	-	Dependent variable is constant.(zero)			
31	-	Dependent variable is constant.(zero)			
32	x	-8.668	0.1048	-	0.026
33	w	-7.736	0.0773	-	0.539
34	w	-0.5198	-0.0126	-	0.371
35	w	-0.1699	-0.0209	-	0.284
36	m	709	0.0683	10380.7	0.826
37	m	1139.2	0.289	3941.9	0.750
38	w	-16.6556	0.1694	-	0.852
39	O	78.7	0.4327	-	0.795
40	m	1685.6	0.3069	5492.3	0.904
41	m	926.8	0.1233	7516.6	0.652
43	-	Dependent variable is constant.(zero)			
44	m	1085.6	0.0754	14397.9	0.85
45	w	-11.4107	0.1234	-	0.327
46	w	1.4147	-0.0313	-	0.078
47	m	975	0.026	37500.0	0.851
48	m	709.8	0.0193	36777.2	0.898
49	p	111.4	0.556	-	0.801
50	w	-4.5627	0.0433	-	0.671
51	w	0.9746	-0.0395	-	0.14
52	-	Dependent variable is constant.(zero)			
53	x	3.0207	-0.0578	-	0.244
57	p	16273.2	0.3437	-	0.741
59	w	-3.6913	0.0345	-	0.005
61	-	Dependent variable is constant.(zero)			
63	w	7.0584	-0.087	-	0.558
64	w	7.4282	-0.0918	-	0.68

Type of Equation

type m 1986-2004
fdepr = f(D1, D2, D3, netfstk[1])

Type o 1988-2004
fdepr = f(D1, D2, D3, sum(iprr[1] + … + iprr[3])

Type p 1988-2004
fdepr = f(D1, D2, D3, timet, sum(iprr[1] + … + iprr[3])

type w 1988-2004
rdepr =fdepr/sum(iprr[1] + … + iprr[3])
log(rdepr) = f(D1, D2, D3, timet)

type x 1990-2004
rdepr =fdepr/sum(iprr[1] + … + iprr[5])
log(rdepr) = f(D2, D3, timet)

Definition of Variables

fdepr: Depreciation in real terms of 2000

netfstk: Cumlative sum of net investment in real terms of 2000

iprr: Gross investment in real terms of 2000

D1: Dummy variable; 1 for 1986, 87, 88, 89

D2: Dummy variable; 1 for 1991, 92, 93, 94

D3: Dummy variable; 1 for 1996, 97, 98, 99

timet: time trend

Estimation criteria
Except for type w and type x
Parameter a > 0 Parameter b > 0 and less than 1
RBSQ > 0.6