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# **A Dynamic GLO Factor Demand Model For DANDY**

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## 1. Introduction

The background for building the Danish DANDY model is that analysis of the economic effects of environmental policies in the direction of a more sustainable economy requires a large scale macroeconomic model preferably based on input output. Some of the environmental goals that an economy such as the Danish have or should have can only be achieved after quite substantial structural changes. We need a substitution away from the most polluting processes towards more environmentally friendly use of the resources. To achieve this it will be necessary to invoke considerable changes in certain consumption and production processes. Some of such changes could be introduced in the model by heavy taxation or the creation of markets for pollution permits. For the model to handle such economic incentives to e.g. substitute away from the use of energy and energy intensive productions and products towards labour, which is less polluting but probably, more costly, it must be able to describe the substitution and the change in substitution effects over time.

For private consumption there is a system such as PADS that will ensure that if the relative price of some of the consumption goods increases there will be a substitution away from these goods towards less expensive ones. This is done in a consistent manner that will secure that the total budget is not exceeded. Something similar is needed for the production side of the model. The best way to do it here is probably a full system of factor demands.

Therefore the following paper is devoted to a description of the first parts of such a factor demand system.

## 2. Theoretical starting point

In order to describe the theoretical background for working with production, it is common to say that companies maximise their profits. Here we will start by assuming that companies minimise their costs, which is a necessary first, but not necessarily sufficient, step on the road to profit maximisation. We will look at the factor demands as contingent on the production  $Y$ . Thus we will focus on how firms will seek to minimise their costs of production factors given a certain level of production.

We start out with the following production function

$$Y = F(K, L, E, M) \tag{1}$$

Thus capital, labour, energy and materials are used to produce  $Y$

The total costs of this production is

$$C = P_K K + P_L L + P_E E + P_M M \tag{2}$$

Now we want to find the levels of the input factors  $K, L, E, M$  that minimizes the costs of production. This will in general give us the following equations

$$\begin{aligned} K^* &= K(Y, P_K, P_L, P_E, P_M) \\ &: \\ M^* &= M(Y, P_K, P_L, P_E, P_M) \end{aligned} \tag{3}$$

Now if we take (2) and define the minimum costs as the costs the industry has, given that the factors of production has adapted to their optimal (cost minimizing) levels we get

$$C^* = P_K K^* + P_L L^* + P_E E^* + P_M M^* = C^*(Y, P_K, P_L, P_E, P_M) \quad (4)$$

Now we see that the minimum costs only depend on the production level and the factor prices. This is so, because  $K^*, L^*, E^*, M^*$  disappears when (3) is substituted into (4).

For a number of functional forms (1) and (4) is called *dual*, since they contain exactly the same information and describe the production technology (the isoquants) to the same extent.

The reason that we are interested in the optimal cost function is that by using Shepards lemma that says, that *the cost-minimising input vector is just given by the derivatives of the of the cost function with respect to the prices*, we can get the factor demand equations

$$K^* = K^*(Y, P_K, P_L, P_E, P_M) = \frac{dC^*(Y, P_K, P_L, P_E, P_M)}{dP_K}$$

:

$$M^* = M^*(Y, P_K, P_L, P_E, P_M) = \frac{dC^*(Y, P_K, P_L, P_E, P_M)}{dP_M} \quad (5)$$

Thus the cost function should be differentiated with respect to the price of the factor in question. So we see that by using Shepards lemma on the costfunctions we can get the same functions as can be derived from the production function (3). This is due to the duality.

The good thing about starting with the cost function is that it is often a lot eaiser to derive the factor demand equation from, compared to the production function, where you would often have so solve some nasty first order conditions.

#### *Choice of functional form*

In order to estimate the equations (5) we must first choose a functional form that can contain these equations. There are a lot of such functional forms available on the market, some of which are CES, Translog and Generalized Leontief. There are also a lot of aspects embedded in the discussion of which functional form to choose.

For instance to my knowledge only the CES function is globally consistent, meaning that the isoquants will not cross the axes and one will not get negative demand for any factor no matter how much the relative prices changes. Investigations by (Thomsen, 1999) show that both Translog and GLO can be shown to behave badly for some combinations of the elasticity of substitution and change in relative prices. Translog behaves badly for low values of substitution (0.5 and below) and both GLO and translog for values of the elasticity of substitution of about 2 and above.

I have tried to work some with the Translog system based on papers by among others Urga (1999), but never really succeeded. Probably because a Translog system – especially if some dynamics is attached to it – requires a LOT of parameters to be estimated. It probably would be the best to use quarterly data for such an exercise. So based on that experience and the relatively bad curvature behavior of the

isoquants of the Translog system around the substitution elasticities of 0.5 or lower (Thomsen, 1999), I ended up choosing the GLO system.

*The GLO system*

The cost function of a GLO system with 4 factors has 16 terms

$$C^* = Y \left[ \mathbf{b}_{KK} P_K^{0.5} P_K^{0.5} + \mathbf{b}_{KL} P_K^{0.5} P_L^{0.5} + \mathbf{b}_{LK} P_L^{0.5} P_K^{0.5} + \dots + \mathbf{b}_{MM} P_M^{0.5} P_M^{0.5} \right] \quad (6)$$

All the 12 off-diagonal parameters ( $\beta_{LK}$  and  $\beta_{KL}$  etc.) are pairwise equal – or they have the same regressors so they should not be identified as different. In an unrestricted estimation they will show up quite different, but there is nothing that can justify such behavior, as they have the same regressors). So that brings us down to 10 parameters to be estimated.

According to Shepards Lemma the factor demand equation can be obtained by differentiating this cost function with respect to the factor prices to get

$$\begin{aligned} K^* &= Y \left[ \mathbf{b}_{KK} + (\mathbf{b}_{KL} P_L^{0.5} + \mathbf{b}_{KE} P_E^{0.5} + \mathbf{b}_{KM} P_M^{0.5}) P_K^{-0.5} \right] \\ &: \\ M^* &= Y \left[ \mathbf{b}_{MM} + (\mathbf{b}_{KM} P_K^{0.5} + \mathbf{b}_{LM} P_L^{0.5} + \mathbf{b}_{EM} P_E^{0.5}) P_M^{-0.5} \right] \end{aligned} \quad (7)$$

The parameter on the diagonal  $\mathbf{b}_{ij}$  can be thought of as the "minimum io-coefficient" which the consumption cannot go under. So they should not be negative. If one or more of the  $\mathbf{b}_{ij}$ 's gets to be negative, we have a problem, because a large change in the relative prices could drive the consumption negative. Therefore if all  $\mathbf{b}$ 's are positive we are sure to have a globally consistent system.

This system has some easy-to-calculate price elasticities. They come as the result of differentiating each of the factor demand equations with respect to the 4 prices and then multiplying by the price over the level of the factor in question. That is a collection of 16 elasticities of which some examples are

$$e_{KK} = 0.5 \mathbf{b}_{KK} \left[ \frac{K^*}{Y} \right] - 0.5 \quad (8)$$

$$e_{KL} = 0.5 \mathbf{b}_{KL} \left[ \frac{K^*}{Y} \right]^{-1} \left( \frac{P_L}{P_K} \right)^{0.5} \quad (9)$$

The  $e_{ij}$  's tell us about possible complementarity ( $e_{ij} < 0$ ) between the factors. It is very often seen that capital and energy are complements, meaning that if the capital input must be increased then energy must also be increased in order to keep the machines running, at least in the short run. The system will be consistent, however, even with some negative  $e_{ij}$  's as long as the relative prices do not change too much.

Now it would be nice to see what we can get out of this framework. But first we need some data to estimate on.

### 3. The Data

For the final estimations of a set of Danish factor demand equations we need 10 different time series, namely the four factors in fixed 1995 prices and their corresponding prices. Also we need the total production value in 1995 prices and the corresponding price of production. Such data can be found more or less entirely in the Danish national accounts, which is already a part of the VAM database for the DANDY model. The model has 52 sectors and it has been the intention to try to model the factor demands in all of these sectors. For the purpose of this paper, however, it has been chosen to use an aggregation of the 13 manufacturing sectors for the estimations<sup>1</sup>. These 13 sectors account for about 25% of the total output. An overview of the estimation database is given in table 1.

**Table 1. Overview of data for the XX aggregate of the 13 manufacturing sectors.**

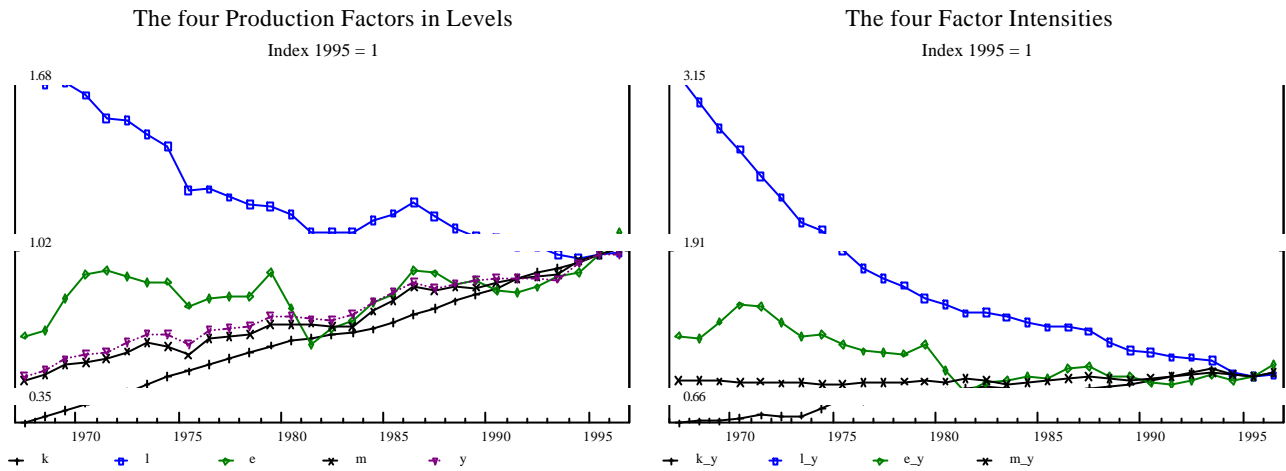
Name	Description	Unit of measurement
QY	Total output in fixed prices, obtained as an aggregate of the 13 manufacturing sectors in the output vector in the VAM file	Mill. DKK
PY	Price of Y, obtained as the implicit price index from dividing the current price output in the XX aggregate by the aggregate in fixed prices. Both found in the vam file.	Index. 1995=1.
QK	The capital is the gross capital stock, since in contrast to the net capital stock it represents the actual productive capacity of the stock. Multiplied by the 1995 value of usercost (before indexing it) gives the current years use of capital in fixed prices.	Mill. DKK
PK	This is by far the most complicated of the data needs. See section x.x for an in depth description of how to derive it. Usercost series lies between 0 and about 0.12, so it has been indexed to match the other prices.	Index 1995=1.
QL	The labour variable consists of the total amount of hours used by the XX sector to produce its output. All the years are multiplied by the 1995 hourly wage to give the fixed price value of labour costs in every year.	Mill. DKK
PL	The hourly wages are indexed by the 1995 value, so PL is equal to one in 1995	Index 1995 = 1
QE	Extracting the energy producing and energy converting sectors from the columns of the IO table obtain the energy variable.	Mill. DKK
PE	The implicit price index obtained by dividing the current and fixed price values for QE.	Index 1995=1.
QM	Materials, which is the amount of input into production. Obtained by summing the columns of the IO tables, subtracting the energy QE, and then aggregating into the XX sector.	Mill. DKK
PM	The implicit price index obtained by dividing the current and fixed price values for QM.	Index 1995 = 1

The derivation of the price on the use of capital PK or the user cost is described in appendix 1. It is derived as the opportunity cost of holding a unit of capital for one year. It is a rather complex variable that embodies the price on new investment, the depreciation rate, the interest rate, the tax rate and other variables.

But lets take a little look at how these variables have developed in the past 30 years or so. We see the amounts of the factors and the prices, both in levels and compared to the production QY.

<sup>1</sup> Actually, none of the 13 sectors performed better alone than do the aggregate.

**Figure 1. The four Production factors**

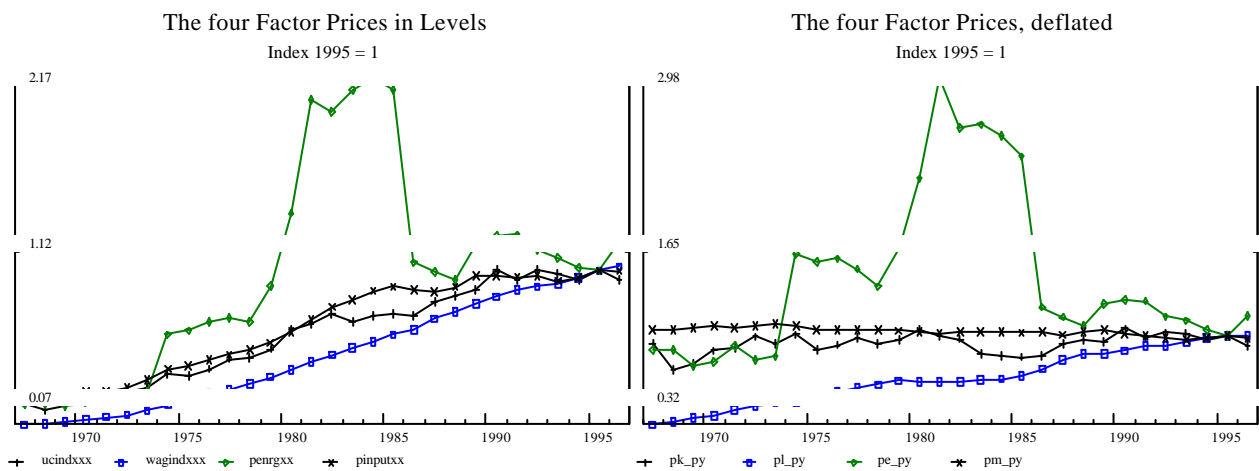


Thus, the data are scaled so that units of measurement in all four factors sum to something less than QY in the base year 1995 both in fixed and current prices. A little margin between Y and the sum of the factors is left for the capital costs of buildings and for profits. This is also true for current prices outside the base year, but NOT for fixed prices. This is due to the fact that all factor prices evolve differently over time reflecting e.g. growth in real wages.

It is very clear that there have been a tremendous decrease in the labour input into this sector. Labour input measured in hours has decreased by 40% over the period, while at the same time output has increased by 88%. That is of course also reflected in the intensity of the labour, which has dropped by a factor 3. The capital intensity have been smoothly increasing during this 30 year period.

The energy intensity is the most time varying component with a slight decreasing tendency. It can be seen that the energy intensity dropped under the oil crises in 1973/74 and also 1979/80. Since 1980 this intensity has been quite stable. The intensity of materials is very stable over time.

**Figure 2. The Factor prices in levels and deflated with PY**



In figure 2 we can see that the real usercost PK/PY has maintained approximately the same level over time with some volatility due to, among other things, the development of the interest rate and the expected inflation in the investment price. The right hand side of the diagram also shows that real wages has increased significantly by more than a factor 3.

When we look at the factor prices it is no wonder that the use of energy is the most volatile of the four factors. Around the oil crises we saw huge increases in the prices of energy. The real price on materials is very stable.

So with these data we will go for a first round of estimates of the system.

#### 4. Estimates, first round

The program TSP is used for the estimation of this system. At this stage G7 could have been used, but at later stages I found that it was easier to use TSP. The program code can be seen in appendix 2, but it is not very pleasant reading.

By using the LSQ order in TSP the set of equations (7) above are estimated. Notice that both sides of the equations are divided by Y to model the factor intensities. The LSQ order is the "general non-linear least squares multiequation estimator". It uses the maximum likelihood. But since these equations are perfectly linear, similar results could have been obtained by "Seemingly Unrelated Regression" (SUR) or 2 stage least squares.

Since it can be rather difficult in some situations to make the estimation converge, it helps quite a lot to set the parameters to some reasonable values rather than just zeros. In this case one would have a good idea about the size of the parameters (bii's) since they express the share of the factor in question relative to Y. But if in other cases one does not have any ideas the parameters one might have a better idea about the size of of the resulting elasticities. Such knowledge is imposed on the procedure by equations like the following

```
set bkk=qk(1996:1)/qy(1996:1)*(1+2* (-0.2) );
```

where the right hand side is just a reformulation of of the own price elasticity formula (8). In this case the number -0.2 is expected to be a reasonable guess of the own price elasticity of capital.

Here are the first results.

**Table 2a. Results from static estimation of GLO model**

Parameter	Estimate	Standard		
		Error	t-statistic	P-value
BKL	.055833	.362697E-02	15.3938	[.000]
BKM	-.047580	.349667E-02	-13.6072	[.000]
BKK	.048505	.540027E-02	8.98196	[.000]
BKE	.429648E-02	.120811E-02	3.55637	[.000]
BLM	.326830	.601780E-02	54.3106	[.000]
BLL	-.069276	.011555	-5.99546	[.000]
BLE	-.010596	.225837E-02	-4.69195	[.000]
BEE	.027723	.204280E-02	13.5708	[.000]
BEM	.012012	.267548E-02	4.48968	[.000]
BMM	.353630	.866207E-02	40.8251	[.000]

**Table 2b. Static estimation of GLO model**

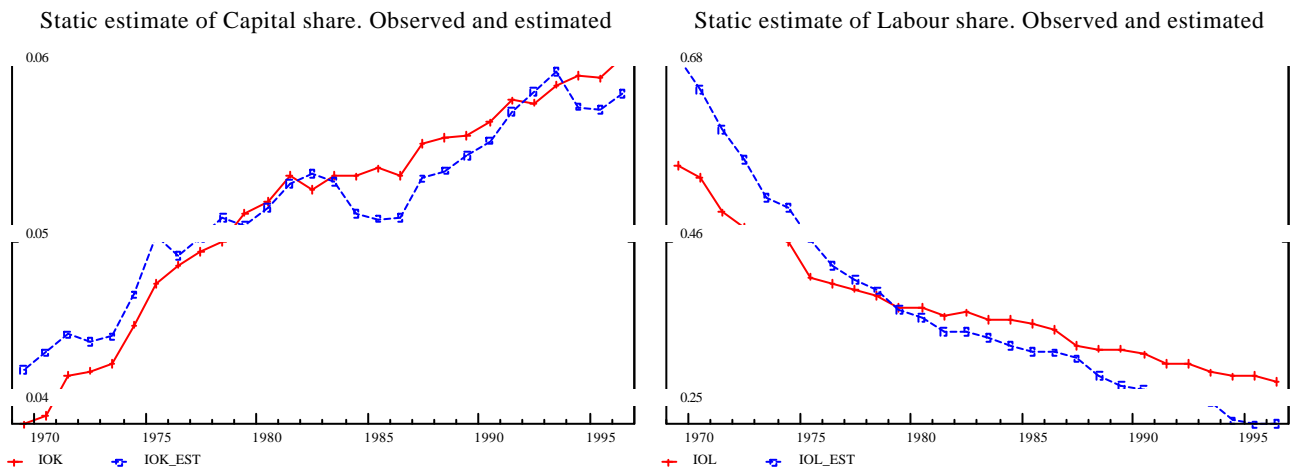
Equation	Variable	Mean	R-squared	DW
E <sub>q</sub> iok	Iok	0.0514	0.95	0.33
E <sub>q</sub> iol	Iol	0.3859	0.99	0.06
E <sub>q</sub> ioe	Ioe	0.0341	0.76	0.33
E <sub>q</sub> iom	Iom	0.5930	0.63	0.11

**Table 2c. Static estimation of GLO model. Long run elasticities.**

	$P_K$	$P_L$	$P_E$	$P_M$
<b>K</b>	-0.11	0.46	0.04	-0.39
<b>L</b>	0.09	-0.62	-0.02	0.55
<b>E</b>	0.06	-0.15	-0.08	0.17
<b>M</b>	-0.04	0.25	0.01	-0.23

We can see that the means in table 2b does not sum to one. Only in the base year 1995 will the four variables iok-m sum to one. This is due to the different development in prices between the series as mentioned earlier. It should be obvious that there are some problems in this estimation. Three of the ten parameters have a negative sign, meaning that the model is not consistent. That is to say, that we are in for negative demands for one or more of the factors when certain conditions are present. The elasticities that sum to one across the row, are generally not as one would expect them to be. The  $e_{KK}$  is quite small and some off-diagonal elements other than the expected  $e_{KE}$  and  $e_{EK}$  are negative. Furthermore, when we look at the graphs, we see that this is clearly not desirable

**Figure 3. Static estimates of capital and labour share of input in production**

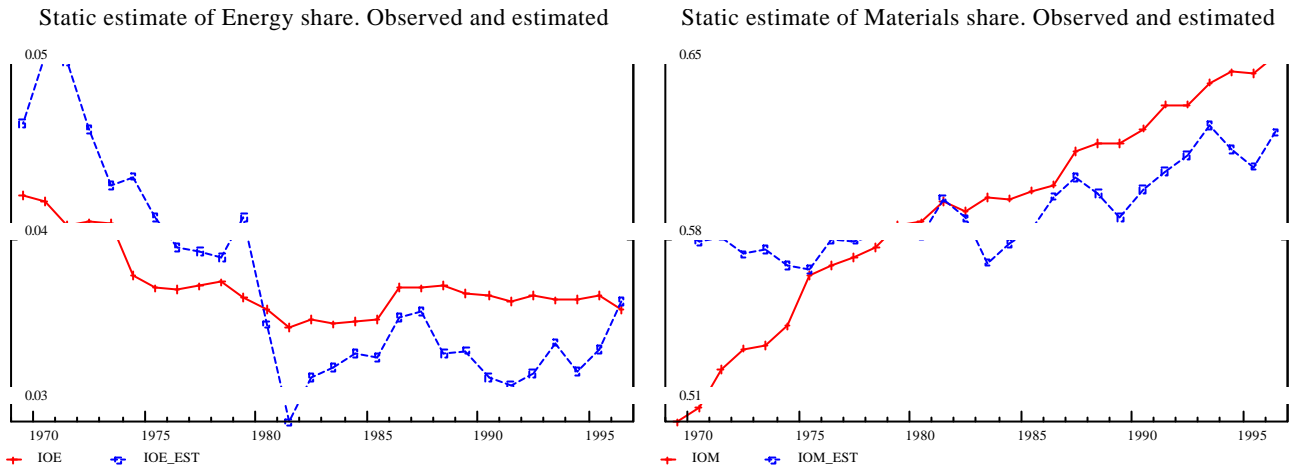


The figures tell the story again; this is not good. The capital estimation seems fairly good compared to the labour which completely misses the point. The estimate is not able to explain the steep fall in the labour share of input into production.

The graphs of energy and materials are a similar sight



**Figure 3. Static estimates of capital and labour share of input in production**



For energy the model has severe difficulties dealing with the fast substitution away from energy around 1980, and in the fourth graph it can be seen, that the model wants to have a considerable increase in the share of materials in relation to total input because it the price of it relative to labour is constantly decreasing. So what can we do to improve on these estimates?

### 5. Time trends

It is very obvious that the equations above is screaming for some time trends to explain e.g. the development in labour productivity. So we must introduce that in the equations. One could start by putting in time in (1) to get

$$Y = G(K, L, E, M, t) \tag{11}$$

There are various ways to get  $t$  into the equations, but here we will stick to a rather simple solution. We simply replace the  $\beta_{ii}$  's by the expression  $\beta_{ii} + z_i * t + zz_i * t^2$  where  $t$  of course is a time variable. This means that we can no longer interpret the  $\beta_{ii}$  's directly as the shares. Also the price elasticities need to be changed now. If we slip  $z_i * t + zz_i * t^2$  into the factor demand equations (7) and differentiate them again, multiply by the price over the level of the factor, and then rearrange the terms, we find that the cross price elasticities (9) are left unaltered, but the own price elasticities (8) now have turned into

$$e_{ii} = 0.5(\mathbf{b}_{ii} + z_i * t + zz_i * t^2) \left[ \frac{i^*}{Y} \right] - 0.5, i = K, L, E, M \tag{12}$$

### 6. Separability

Another thing that is often discussed in relation to factor demand systems is whether the estimation should be nested such that the division between a fraction of the factors is determined before the complete system is estimated. As an example one could imagine that the the factors were nested ((KLE)M). That means that at first the division between K, L and E is decided upon. Then the division between this KLE aggregate and M is determined. With such a structure one would say that the

function is weakly *separable* in M. In practice this means that the price  $P_M$  not affects the relative distribution between K, L and E, but it could very well affect the levels of them.

Apart from the symmetry restrictions we could consider to lower the number of parameters to be estimated even further since it is often difficult to estimate this system even with 10 parameters. Therefore we could impose some separability restrictions on the parameters, meaning that if M is weakly separable, changes in the price on materials will affect the other factors with equal strength (equal to the elasticity), so an increase in the price of materials will not affect the COMPOSITION of the other factors. Such a separability assumption that will save some parameters, can be imposed by making sure that the cross price elasticities in (9)

$$e_{KM} = e_{LM} = e_{EM} \tag{10}$$

are equal when the prices  $P_K$  to  $P_M$  have the 1996 value. Since the elasticities depend on the relative factor prices, this restriction cannot be maintained for all values of the explanatory variables over time. So the restriction can only be imposed for one year. By using (10) one can figure out functions for  $\beta_{KM}$  and  $\beta_{LM}$  where they are functions of other prices and betas in order to make them equal to  $\beta_{EM}$ . If  $\beta_{EM}$  is put equal to zero  $\beta_{KM}$  and  $\beta_{LM}$  will also be zero. In that case we say that M is strongly separable. In the same way one can figure out the conditions to make other nesting schemes like for instance ((KL)E)M) or (((KE)L)M). As we shall see later some degree of separability can help a lot on the easiness of estimation.

## 7. Estimates, second round

In the next set of estimates, our two new enhancements of the model, namely the time trends and the weak separability of the materials ( a ((KLE)M) nesting) will be implementet. Furthermore two minor things are changed. One is that we now only have the  $i^*$  as the dependent variable and Y is multiplied on the right hand side. That opens for the possibility to take logarithms of the dependent variable and on the entire right hand side at the same time. Now the results look like this

**Table 3. Static estimation of GLO model with time trends and M weakly separable ((KLE)M)**

Parameter	Estimate	Standard Error	t-statistic	P-value
BKL	.010540	.010180	1.03542	[.300]
BLE	.840646E-02	.515945E-02	1.62933	[.103]
BKE	.114567E-02	.147060E-02	.779046	[.436]
BEM	-.484169E-03	.382894E-02	-.126450	[.899]
BKK	.049074	.680491E-02	7.21152	[.000]
BEE	.022117	.336049E-02	6.58149	[.000]
ZK	.658407E-03	.222879E-03	2.95410	[.003]
ZZK	.767653E-06	.846997E-05	.090632	[.928]
BLL	.243346	.022650	10.7436	[.000]
ZL	-.106034E-02	.132672E-02	-.799224	[.424]
ZZL	.493845E-03	.600501E-04	8.22388	[.000]
ZE	.822433E-04	.220272E-03	.373372	[.709]
ZZE	.290715E-04	.778995E-05	3.73193	[.000]
BMM	.625263	.041083	15.2197	[.000]
ZM	.365100E-02	.637512E-03	5.72696	[.000]
ZZM	.806507E-04	.230937E-04	3.49233	[.000]

In the results from the second round of estimates we see that there are only 8  $b_{ii}$  's left to be estimated, but now we have in addition four sets of two time trend variables that must also be estimated. Now only one of the b's is negative, so that is an improvement.

**Table 2b. Static estimation of GLO model with time trends and M weakly separable ((KLE)M)**

Equation	Variable	Mean	R-squared	DW
leqk	lak	9.758	0.99	0.70
leql	lal	11.737	0.91	0.30
leqe	lae	9.342	0.75	1.53
leqm	lam	12.210	0.99	1.39

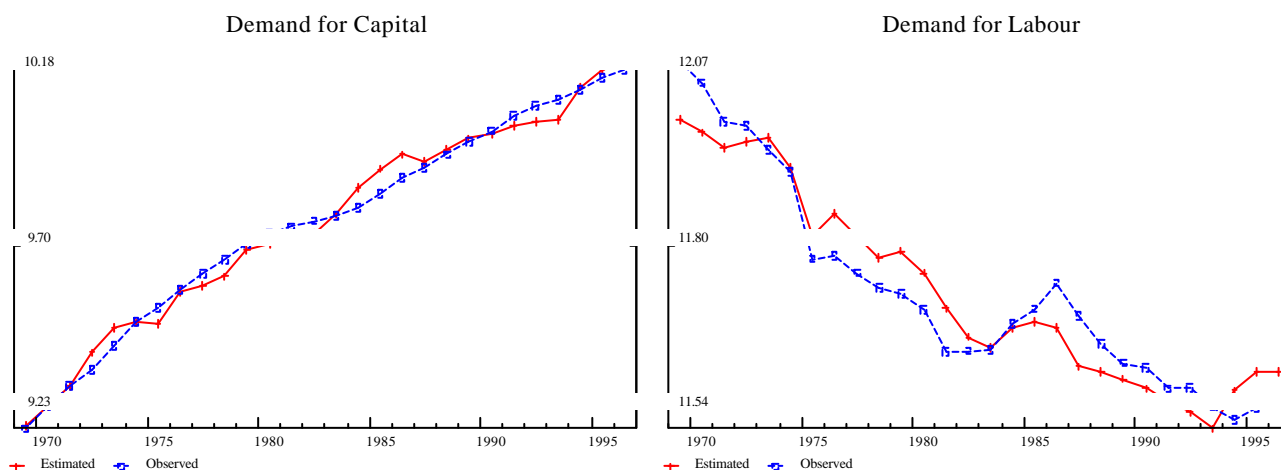
**Table 2c. Static estimation of GLO model with time trends and M weakly separable ((KLE)M). Long run elasticities.**

	$P_K$	$P_L$	$P_E$	$P_M$
<b>K</b>	-0.09	0.09	0.01	-0.01
<b>L</b>	0.02	-0.03	0.02	-0.01
<b>E</b>	0.02	0.13	-0.14	-0.01
<b>M</b>	0.00	0.00	0.00	-0.00

We can see that the misspecification of the model as indicated by the very low DW statistics in the first simulation has improved for all four factors, but especially much for energy and materials. Notice now that that the separability constraint has made the three elasticities  $e_{KM}$ ,  $e_{LM}$  and  $e_{EM}$  equal. In general the elasticities are quite low here, but at least the have the right signs.

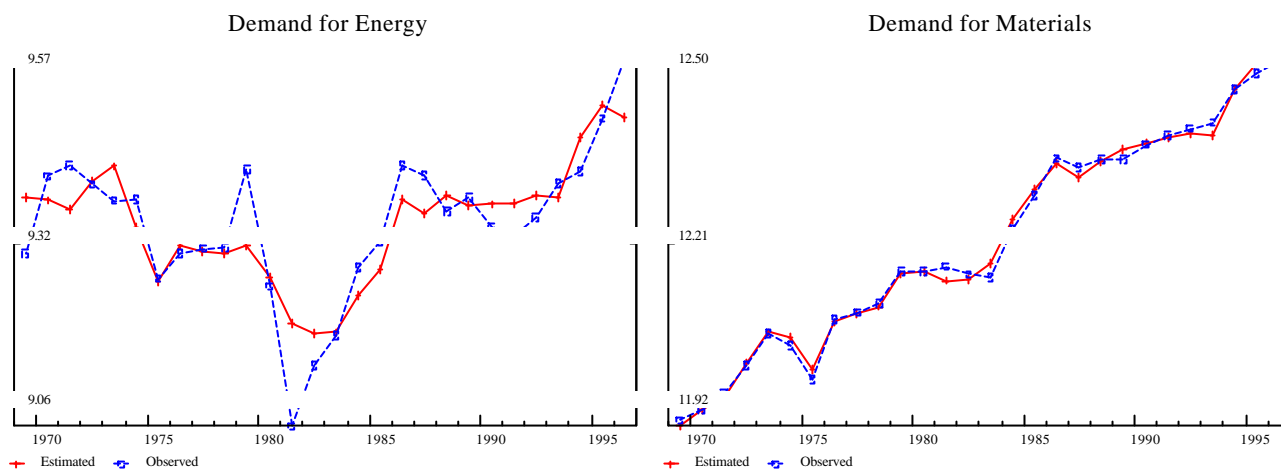
The graphical display of the results also show considerable improvements

**Figure 3. Static estimates of the demand for capital and labour in production.** With time trends and M weakly separable ((KLE)M).



Now we see that we have got some more reasonable fits. The graphs are not systematically wrong as was the case in the previous estimate. Apparently the capital estimate fits quite well, but that should not fool one. When the results are used for calculating investment on the basis of the changes in stock that can be derived from these estimates we may still see some substantial misses on the observed investment.

**Figure 4. Static estimates of the demand for energy and materials in production.** With time trends and M weakly separable ((KLE)M).



In the case of energy we are doing pretty much better now. The estimation did not catch all of the steep decline in energy demand in 1979 / 1980 but we are doing better than before. Materials are pretty close to the observed values. Now we must remember that these are static estimates and by that we assume that the demand for the factors will adapt to their long run level in the very same year. There is so much evidence that says that this is not true, especially for the capital stock. If e.g. an increase in the general demand increases production  $Y$  by 5% the demand for input will increase by 5% since we do assume here that there are constant returns to scale. So the demand for capital will increase by 5% in the first year, but investment takes time. So the most likely case is that it will take a number of years before the capital stock has adapted to this demand shock. Even labour might need some time to adapt due to labour hoarding effects. In the next section we will specify the model in a dynamic way so as to allow for an adaptation over time.

## 8. Dynamic specification

There are various more or less sophisticated ways to introduce dynamics into these equations. In figure 5 below there is an illustration of different ways in general to think of the adaptation process. If we use only the static estimation as above we will go from A to C in one single step. The so-called "second generation model" will adapt over time along the path between A and C. But it can be seen that we are on the wrong isoquants for a period of some years. The "third generation model" on the contrary, will go directly to the correct isoquant, but with a wrong combination of factors. Because capital is slow to adapt, other factors like materials or maybe labour must compensate for this in the short run. So they will overreact. That is why we talk about quasi-fixed contrary to flexible factors.

Thomsen(2000) present a short cut to reach a third-generation model quite easily. He is using the long run function to derive the short run cost function. This is not possible, however, to do analytically with the Translog and the CES forms. But with the long run GLO it is possible. The trick is first to solve the quasi fixed factor equations for their own prices. Now this price is called a *virtual shadow price*, and when it is substituted into the equations for the flexible factors, they will react according to those prices and make the sufficient compensation for the slowness in the capital adaptation. This system, however, can be difficult to estimate and to get to converge, so in this paper we will look at the second generation model.

**Figure 4a. Difference between second and third generation models**

This kind of dynamic behaviour can be introduced by through a standard error-correction model, here for the capital equation

$$D \log(K) = I_1 D \log(K^*) + I_2 [\log(K_{-1}^*) - \log(K_{-1})] \quad (11)$$

K	the observed capital stock
K*	the estimated long run value of the capital stock
D	first differences
log	logarithms
$\lambda_1, \lambda_2$	parameters

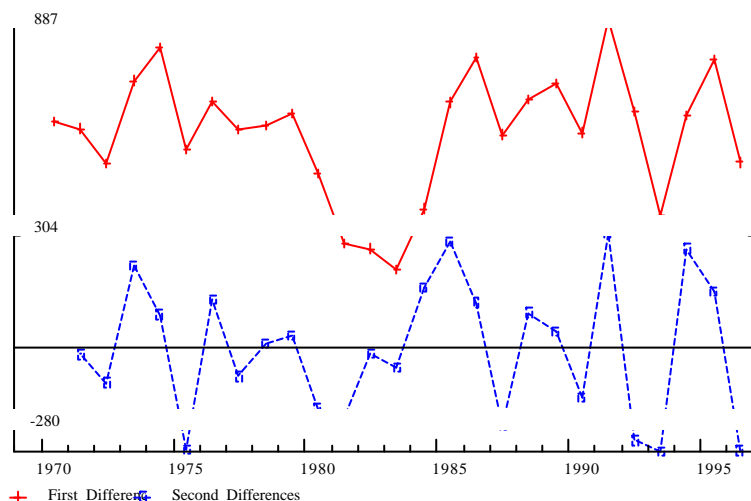
The parameter  $\lambda_1$  gives us the first year effect, while  $\lambda_2$  tell us about the speed of adjustment towards the new equilibrium. They most both lie in the interval from zero to one. Thus if  $\lambda_1$  is equal to one, the entire adaptation towards the long run level will take place in the first year. If  $\lambda_1 = \lambda_2$  we have a pure partial model and so (11) can be seen as a generalized partial adaptation model.

For the modelling purpose it is convenient not to have to work with these first differences, so we can reparametrise (11) to get an equation in levels (see e.g. Johnston and Dinardo, 1997)

$$\log(K) = I_1 \log(K^*) + (I_2 - I_1) \log(K_{-1}^*) + (1 - I_2) \log(K_{-1}) \quad (13)$$

which will give us the exactly the same results as by using 11, just in a more convenient way. As we shall see shortly, such a formulation works particularly fine with energy and materials and to some extent also Labour. But for Capital we do not get all the autocorrelation out. It is well know that capital is normally not an I(1) variable meaning that it is stationary, when first differences are taken, but an I(2) variable. That means that we have to take the second differences to get rid of the trend as well as it can be seen in this figure

**Figure 5**  
Demand for capital, first and second differences



To take care of that in the model specification we have a number of possibilities. We could put K and K\* lagged two times in or we could put in the dependent variable lagged once. That means the dependent variable in (11). This can be done in an equation such as (13) but with other parameter restrictions and introducing  $K_{-2}$ . This construction really help on the estimation in terms of fit and test statistics. But it turns out that such a construction gives an overshooting in in the estimated capital demand meaning that if  $K^*$  is raised permanently by 1% then the model will start to adapt K gradually to this new level, but in a number of years in the period between year 1 and the the where K is fully adapted, K will lie above one.

What turns out to work best, is a rho-construction, which also could be called an AR(1) term. In principle one introduces the lagged residuals into the model with a parameter  $\rho_K$  in front. This can also be included in the model (13). In that case we need to introduce also  $K_{-2}$  and  $K_{-2}^*$  in the equation and apply the right parameterization to the equation

$$\log(K) = I_1 \log(K^*) + (I_2 - I_1 - rI_1) \log(K_{-1}^*) - r(I_2 - I_1) \log(K_{-2}^*) + (1 - I_2 + r) \log(K_{-1}) - r(1 - I_2) \log(K_{-2}) \quad (14)$$

This in implemented in TSP in the following way

```
frml dynk log(K) =
                    11k * log(Kstar)
                    +(12k-11k-(rhok*11k)) * log(Kstar(-1))
                    -rhok*(12k-11k) * log(Kstar(-2))
                    +(1-12k+rhok) * log(K(-1))
                    -rhok*(1-12k) * log(K(-2));
```

The variable and parameter names here are made up to be close to (14), but the ones that are necessary for the model can be seen in Appendix 2.

### 9. Estimates, final round

The model can be solved in two ways now. We have already estimated the equations determining the long run variables or the "desired" level of the variables. Now we could take those estimates and put them into the dynamic equations (13) and (14) to get the short run or estimated value of the capital stock. This would be called the Engle and Granger two step method to solving error correction models (Engle and Granger, 1987). However, one could also estimate the system of long run equations simultaneously with the short run estimates to get the overall performance of this system as good as possible. Such a procedure produces estimates as are shown in table 3

**Table 3. Dynamic estimation of GLO model with time trends and M weakly separable ((KLE)M)**

Parameter	Estimate	Standard Error	t-statistic	P-value
BKL	.046586	.035033	1.32975	[.184]
BLE	.013625	.748643E-02	1.82001	[.069]
BKE	-.034873	.765302E-02	-4.55673	[.000]
BEM	.033338	.555029E-02	6.00655	[.000]
BKK	-.365442E-02	.034327	-.106459	[.915]
BEE	.020747	.411828E-02	5.03783	[.000]
ZK	.315081E-03	.116415E-02	.270654	[.787]
ZZK	.128385E-04	.493305E-04	.260254	[.795]
G0K	.070313	.015363	4.57694	[.000]
RHOK	.113297	.202777	.558727	[.576]
G1K	.029433	.016363	1.79870	[.072]
BLL	-.042779	.047888	-.893311	[.372]
ZL	.108516E-02	.189929E-02	.571352	[.568]
ZZL	.275333E-03	.891144E-04	3.08966	[.002]
G0L	.274671	.084288	3.25872	[.001]
G1L	.584858	.077477	7.54879	[.000]
ZE	.538706E-03	.328110E-03	1.64185	[.101]
ZZE	.367714E-04	.122612E-04	2.99901	[.003]
G0E	.736305	.132355	5.56309	[.000]
G1E	.484779	.133961	3.61879	[.000]
BMM	.269322	.058469	4.60626	[.000]
ZM	.755155E-03	.843709E-03	.895042	[.371]
ZZM	.143449E-03	.305853E-04	4.69013	[.000]
G0M	.724659	.155522	4.65954	[.000]
G1M	1.02665	.067015	15.3198	[.000]

**Table 3b. Dynamic estimation of GLO model with time trends and M weakly separable ((KLE)M)**

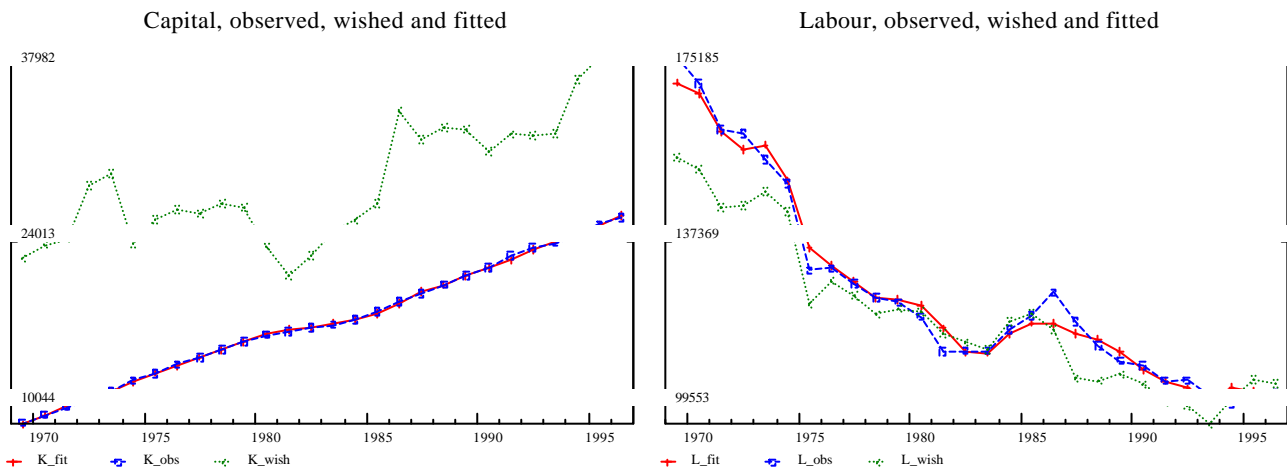
Equation	Variable	Mean	R-squared	DW
prek	qqk	9.758	0.99	1.88
prel	qql	11.737	0.98	1.34
pree	qqe	9.342	0.77	1.65
prem	qqm	12.210	0.99	1.69

**Table 3c. Dynamic estimation of GLO model with time trends and M weakly separable ((KLE)M). Long run elasticities.**

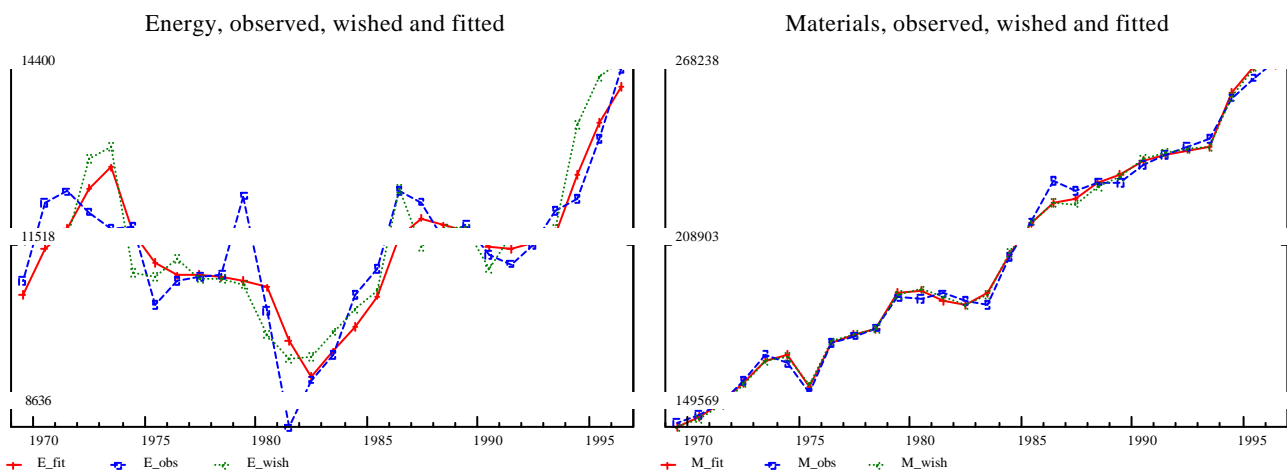
	$P_K$	$P_L$	$P_E$	$P_M$
<b>K</b>	-0.52	0.28	-0.22	0.46
<b>L</b>	0.09	-0.58	0.03	0.46
<b>E</b>	-0.47	0.19	-0.18	0.46
<b>M</b>	0.06	0.19	0.03	-0.28

This is seemingly very good. We have some fairly reasonable elasticities and both DW and R-squared are very good. However, there is one problem, which will show up, when we take a look at the corresponding graphs.

**Figure 5. Static estimates of the demand for capital and labour in production.** With time trends and M weakly separable ((KLE)M).



**Figure 6. Static estimates of the demand for energy and materials in production.** With time trends and M weakly separable ((KLE)M).



We can see from the graphs, that the fits are quite nice now. However, the long run level of capital demand is 100%-150% above the actual level. The fit is good, but the reason is that the parameter  $g_{1k}$ ,



is only 0.029. This parameter gives the first year adaptation of the capital stock to the desired long run level. So only a 3% closing in on the gap in the first year is probably not something that one would want in the model. Contrary to the second round of estimates, the parameter *bkk* is now negative and very insignificant.

The standard deviation on the *g1k* parameter is too high to allow one to tie it to something reasonable.

So something is wrong and something has to be done about it.

## Appendix A

This paper is about the user-cost expression in the model. It needs to be rewritten somewhat to make it more readable. It needs some lists of variables in connection with the equations specified. Although well hidden, it does actually contain a description of the method used.

At the end of the paper I have attached the little G add-file that calculates the expression.

### A user-cost expression for Dandy3m

*Peter Rørmose, Maryland April 2001*

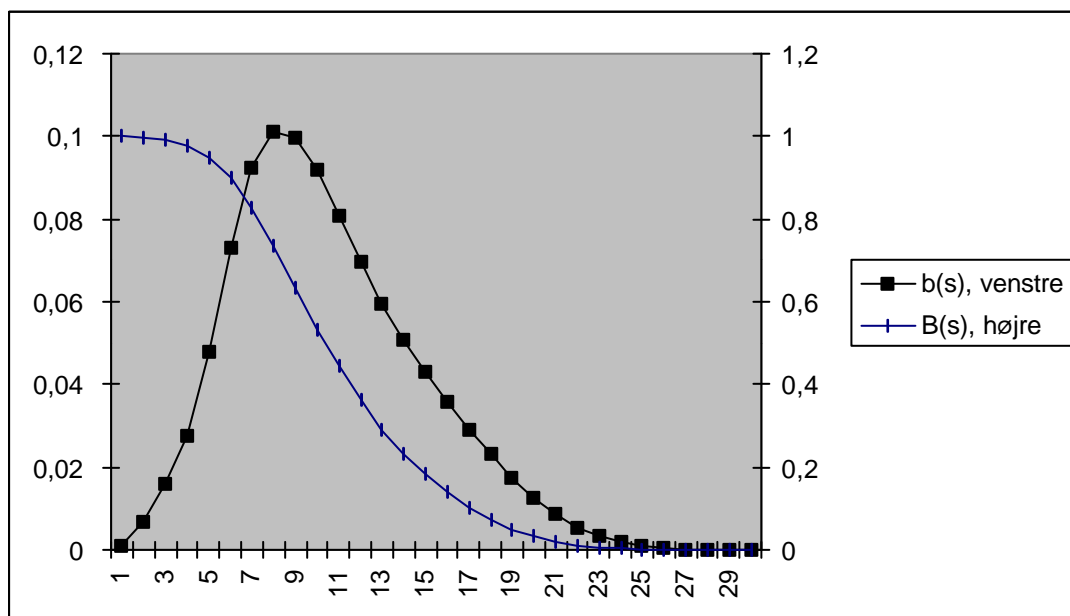
---

For modelling the demand for capital in a factor demand system, we need a price on capital that reflects the service coming from one unit of capital in one year. This is most often measured as the opportunity cost of holding this unit of capital for one year. In its most simple form the user cost can be expressed as

$$c_t = \frac{q_t}{\Phi_0} \tag{1}$$

where  $q_t$  is the investment price, and  $\Phi_0$  is the present value of all the future services from one new unit of capital. Thus usercost is dependent of the survival curve for the underlying investments. The survival curve can be assumed to have one out of a wide variety of different forms. It depends on among other things on the estimated expected lifetime for the investment good in question.

**Figure 1. Scrapping and survival curves  $b(s)$  og  $B(s)$ , Winfrey L2, expected lifetime 10 years**



This figure shows the expected scrapping curve (■ left scale) for an investment good with an expected lifetime of 10 years. This curve is used to create the survival curve (◆ right scale). This is based on

empirical research done by Winfrey some 60 years ago, but the general shape of the curves probably still applies today. Thinking about e.g. a car, it makes a lot of sense to see that in the first years of the life of the car only a couple of percent will be scrapped, this number increasing to approximately when the expected lifetime is reached, and declining with a very long tail to the right, meaning that some cars will be as much as 30 years old.

If we wanted a geometric survival function, one example of a scrapping curve could be a constant at e.g. 10 pct. to give us a smoothly declining survival curve (1, 0.9, 0.81, 0.72 etc.) which, however, in an empirical light is a bit hard to believe in. This is important, because it can be shown that only under a geometrically declining survival curve (1) will resemble the general neoclassical usercost expression

$$c_t = q_t(\mathbf{d} + \mathbf{r}) \quad (2)$$

where  $\rho$  is a discount rate.

It is a fact that the capital stock data, at least from Statistics Denmark, have not been made with a geometrically declining survival curve, so instead it can be shown, that if we define the economic depreciation rate  $\delta_t$  as  $Dn_t / Kn_{t-1}$  we can get the following general formula for usercost

$$c_t = \frac{q_t(\mathbf{d} + \mathbf{r})}{1 + \mathbf{r}} \frac{Kn_t}{K_t} \quad (3)$$

Thus now the relationship between the net and the gross capital stock is multiplied on the general neoclassical expression. The ratio of the net capital stock over the gross capital stock is an indicator of the average age of the capital stock. The newer the stock is in general, the closer the net value is to the gross value. So the older the stock is the lower the usercost will be. The advantage of this expression is that we do not have to know anything about the survival curve of the investments made (in principle they are reflected in the value of  $Kn_t$  and we do not have to make the assumption that it declines geometrically.

In the following the division by  $(1+\rho)$  is skipped<sup>2</sup>, so now it is only a question of discounting the service from the capital in period  $t$ . The discount rate  $\rho$  consists of two terms: an after tax nominal interest rate and a measure of inflation. As a measure of inflation it is suggested to use the growth rate of the investment price  $q_t$ , so now we have

$$c_t = q_t \left( r + \mathbf{d} - \frac{\Delta q_t}{q_{t-1}} \right) \frac{Kn_t}{K_t} \quad (4)$$

So now we have the costs of using one unit of capital equipment is – apart from the price – the forgone rent and the physical depreciation, but on the positive list we now have a possible increase in the price of the investment good. This probably should be the price on the capital good obtainable on the market for used capital goods, but for most goods it is rather difficult to get these prices. If we had the geometric curve it would not matter if we used the price of used or new capital. They would be equal.

---

<sup>2</sup> In accordance with working papers from Statistics Denmark

It can be argued that the relevant measure of inflation is more likely to be an expected rate of inflation, because if the price of capital is expected to rise in the future, so will the capital gains and the usercost will go down. If the price increases, it is expected that there will also be price increases in the future. Such an expectation will last for a number of years. To model this we introduce a geometric lag in the investment price

$$\mathbf{p}_t^e = \alpha \mathbf{p}_{t-1}^e + (1-\alpha) \mathbf{p}_t \quad , \quad \mathbf{p}_t = \frac{\Delta q_t}{q_{t-1}} \quad (5)$$

According to this equation, the expected rate of inflation is a weighed average of last year's expectation and the realised inflation in the current year. Thus it is an adaptiv formation of expectations. The speed of adjustment depends on  $\alpha$ . The bigger  $\alpha$  is, the longer the adjustment to a new equilibrium will take.

The problem about the implementation of this is the lagged expected rate of inflation on the right hand side. So we need to generate some data for this variable. Using a moving average form we can write the model as

$$\mathbf{p}_t^e = (1-\alpha) \mathbf{p}_t^* + \alpha^t \mathbf{p}_0^e, \quad (6)$$

$$\mathbf{p}_t^* = \mathbf{p}_t + \alpha \mathbf{p}_{t-1} + \dots + \alpha^{t-1} \mathbf{p}_1$$

So if we had data for  $\pi^e$  and  $\pi$  we could estimate the parameters  $\alpha$  and  $\pi_0^e$ . But we do not know  $\pi^e$ . What can be done is to determine  $\alpha$  a priori, and then to look for the value of  $\pi_0^e$  that gives the most consistency between the realised and the expected inflation. Letting  $\pi_0^e$  be a parameter in the following regerssion can do it

$$\mathbf{p}_t = (1-\alpha) \mathbf{p}_t^* + \alpha^t \mathbf{p}_0^e + \mathbf{e}_t \quad (7)$$

The predicted value of  $\pi$  from this regression can now be used as a measure for  $\pi^e$ .

But we also need to adjust the user cost expression for payment of taxes and for tax credits on in payments. We need the user cost to be a pre-tax expression, which is the most relevant in an cost minimization problem where other prices are pre-tax variables too. When the factor demand system is estimated the *relative* prices will work as after tax variables anyway. With what we have now and taxes included we get the following formula

$$c_t = \frac{q_t(1-sz)}{(1-s)} \left( (1-s)i + \mathbf{d} - \mathbf{b} \mathbf{p}_t^e \right) \frac{K n_t}{K_t} \quad , \quad \mathbf{p}_t^e = \alpha \mathbf{p}_{t-1}^e + (1-\alpha) \mathbf{p}_t \quad , \quad \mathbf{p}_t = \frac{\Delta q_t}{q_{t-1}} \quad (8)$$

Here we have some new variables. First of all we see that the alternative rent  $r$  has turned into the interest rate term  $(1-s)i$ . This is due to the fact that the other factor prices comes as pre-tax, the minimum cost function is homogenous of degree 1 in all factor prices and thus the factor demand functions are homogenous of degree 0 in those prices.

The variable  $z$  is the present value of future depreciation of one capital unit. Finally the  $\beta$  is a weight given to the expected inflation term. Experiments can be made with that to see what gives the best fits.

### **G7 add-file to calculate user cost of capital**

(The contents behind the variable names are explained in text above)

```
f alpha = 0.25
f t = (time-1995)
f pistar = (0.25/(1-alpha))*rpim+(0.25/(1-alpha))
           *rpim[1]+(0.20/(1-alpha))
           *rpim[2]+(0.15/(1-alpha))
           *rpim[3]+(0.10/(1-alpha))
           *rpim[4]+(0.05/(1-alpha))
           *rpim[5]
f apowt = @exp(t*@log(alpha))
con 100 0.75 = a1
lim 1967 1997
r rpim = !pistar, apowt
f pie = predic

f beta = 0.75
f uc = (1/(1-tsdsu))*pim*(1-(tsdsu*bivm))*
       (((1-tsdsu)*interest)+deprat-(beta*pie))*(fkn/fkg)
```

## Appendix B. TSP program.

```
options crt;
in klem;           ? Reads in databank .tlb
freq a;           ? Frequency of data
smpl 1967,1996;   ? Sample period

*****
set separ= 0;     ? 0=KLEM 1=KLE,M 2=KE,L,M 3=KL,E,M (Weak separability)
set separ1= 0;   ? if M is strongly separable
set separ2= 0;   ? if E is strongly separable
set separ3= 0;   ? if L is strongly separable

set tidk =0;      ? time in qk equation
set tidtidk =0;  ? time squared in qk equation
set tidl =0;     ? time in ql equation
set tidtidl =0;  ? time squared in ql equation
set tide =0;     ? time in qe equation
set tidtide =0;  ? time squared in qe equation
set tidm =0;     ? time in qm equation
set tidtidm =0;  ? time squared in qm equation
*****

genr QY = QY / 1000;
genr QE = QE / 1000;
genr QM = QM / 1000;
genr tid = year - 1995;      ? Generate time variable
set tid_96=tid(1996:1);

dot k l e m;                ? Create parameters. Initial value = 0
  param bk. bl. be. bm.;
  set p._96=p.(1996:1);      ? Create variables for later use in calculation
enddot;                     ? of elasticities

? Give starting values for own-price elasticities
set bkk=qk(1996:1)/qy(1996:1)*(1+2*      (-0.2)      );
set bll=ql(1996:1)/qy(1996:1)*(1+2*      (-0.4)      );
set bee=qe(1996:1)/qy(1996:1)*(1+2*      (-0.1)      );
set bmm=qm(1996:1)/qy(1996:1)*(1+2*      (-0.2)      );

dot k l e m;                ? Divide the variable
  io. = q. / qy;            ?
  lq. = log(q.);
enddot;

dot k l e m;
  if(tid.=1);
  then;
  param z. 0;
  else;
  const z. 0;
  if(tidtid.=1);
  then;
  param zz. 0;
  else;
  const zz. 0;
enddot;

dot k l e m;
  param g0. g1.;
enddot;

param rhok rhol;
?const glk 0.25;

frml eqiok iok=bkk+z*tid+z*tid**2+
  (bkl*p1**0.5+bke*pe**0.5+bkm*pm**0.5)*pk**(-0.5);
frml eqiol iol=bll+z1*tid+z1*tid**2+
  (bkl*pk**0.5+ble*pe**0.5+blm*pm**0.5)*p1**(-0.5);
frml eqioe ioe=bee+z*tid+z*tid**2+
  (bke*pk**0.5+ble*p1**0.5+bem*pm**0.5)*pe**(-0.5);
frml eqiom iom=bmm+z*tid+z*tid**2+
  (bkm*pk**0.5+blm*p1**0.5+bem*pe**0.5)*pm**(-0.5);
```

```
frml eqk ak = qy*(bkk+zk*tid+zzk*tid**2+
  (bkl*pl**0.5+bke*pe**0.5+bkm*pm**0.5)*pk**(-0.5));
frml eql al = qy*(bll+zl*tid+zzl*tid**2+
  (bkl*pk**0.5+ble*pe**0.5+blm*pm**0.5)*pl**(-0.5));
frml eqe ae = qy*(bee+ze*tid+zze*tid**2+
  (bke*pk**0.5+ble*pl**0.5+bem*pm**0.5)*pe**(-0.5));
frml eqm am = qy*(bmm+zm*tid+zzm*tid**2+
  (bkm*pk**0.5+blm*pl**0.5+bem*pe**0.5)*pm**(-0.5));

frml prek qqk = (1-g0k+rhok)*lqk(-1)+
  (-rhok*(1-g0k))*lqk(-2)+
  g1k*lak+
  (g0k-g1k-(rhok*g1k))*lak(-1)+
  (-rhok*(g0k-g1k))*lak(-2);
?frml prel qq1 = (1-g01+rho1)*lq1(-1)+
? (-rho1*(1-g01))*lq1(-2)+
? g11*lal+
? (g01-g11-(rho1*g11))*lal(-1)+
? (-rho1*(g01-g11))*lal(-2);
frml prel qq1 = (1-g01)*lq1(-1)+g11*lal+(g01-g11)*lal(-1);
frml pree qqe = (1-g0e)*lqe(-1)+gle*lae+(g0e-g1e)*lae(-1);
frml prem qqm = (1-g0m)*lqm(-1)+g1m*lam+(g0m-g1m)*lam(-1);

? SEPARABILITY IS APPLIED TO TE RELEVANT PARAMETERS
separab sepkmem seplmem sepklel;
dot k l e m;
  eqsub eqio. sepkmem seplmem sepklel;
  eqsub eq. sepkmem seplmem sepklel;
  eqsub pre. sepkmem seplmem sepklel;
enddot;

? If M is to be seperated out
  if(separ1=1);then;do;
  if(separ=1);then;const bem 0;
  if(separ=2);then;const bem 0;
  if(separ=3);then;const blm 0;
enddo;

? Estimation with ble=0 <=> (K,E) eller (K,L) given M separated out
  if(separ2=1);then;const ble 0;
  if(separ3=1);then;const ble 0;

dot k l e m;
  frml leq. la. = log(a.);
  eqsub leq. eq.;
enddot;

dot k l e m;
  eqsub(noprint)pre. leq.;
enddot;

dot k l e m;
  la. = lq.;
  qq. = lq.;
enddot;

smpl 1969,1996;

lsq(noprint maxit=5000 tol=0.000001 hiter=g wname=own step=golden maxsqz=50
  sqztol=0.01)eqiok eqiol eqioe eqiom;
siml(static,tag=io,endog=(iok,iol,ioe,iom))eqiok eqiol eqioe eqiom;
write iok iol ioe iom;

lsq(noprint maxit=5000 tol=0.000001 hiter=g wname=own step=golden maxsqz=50
  sqztol=0.01)leqk leql leqe leqm;
lsq(noprint maxit=10000 tol=0.0001 hiter=g wname=own step=golden maxsqz=50
  sqztol=0.01)prek prel pree prem;

siml(dynam,tag=_lr,endog=(lak lal lae lam))leqk leql leqe leqm;
kstar=exp(lak_lr);
lstar=exp(lal_lr);
estarc=exp(lae_lr);
mstar=exp(lam_lr);
```

```
set sepkmem;
set seplmem;
set sepklel;

ela e96 ekk ell eee emm ekl eke ekm
                elk ele elm eek eel eem emk eml eme;
dot k l e m;
  dot k l e m;
    write(file='glo.doc'format=labels)e..;
  enddot;
enddot;

write(file='glo.doc',format='(35h Elasticities in 1996                )');
write(file='glo.doc',format='(4f7.2)')e96;

siml(dynam,tag=p,endog=(lak lal lae lam qqk qql qqe qqm))
  leqk leql leqe leqm prek prel pree prem;

write lqk,lql,lqe,lqm;
?*****
? PROCEDURES
?*****
proc separab sepkmem seplmem sepklel;

? Three possible nests

if(separ=0);then;do;
? (1) (K,L,E,M)
  frml sepkmem bkm=1/pkm * pem*bem * (bkk*pkk+bkl*pk1+bke*pke)
    /(bek*pek+bel*pel+bee*pee);
  frml seplmem blm=1/plm * pem*bem * (blk*plk+bll*p11+ble*ple)
    /(bek*pek+bel*pel+bee*pee);
  frml sepklel bkl=bkl;
enddo;

if(separ=1);then;do;
? (1) ((K,L,E),M)
  frml sepkmem bkm=1/pkm * pem*bem * (bkk*pkk+bke*pke)
    /(bke*pek+bee*pee);
  frml seplmem blm=1/plm * pem*bem * (blk*plk+bll*p11+ble*ple)
    /(bke*pek+bee*pee);
  frml sepklel bkl=bkl;
enddo;

if(separ=2);then;do;
? (2) (((K,E),L),M)
  frml sepkmem bkm=1/pkm * bem*pem * (bkk*pkk+bke*pke)
    /(bke*pek+bee*pee);
  frml sepklel bkl=1/pk1 * ble*pel * (bkk*pkk+bke*pke)
    /(bke*pek+bee*pee);
  frml seplmem blm=1/plm * bem*pem * (bkl*plk+bll*p11+ble*ple)
    /(bke*pek+ble*pel+bee*pee);
  eqsub seplmem sepklel;
enddo;

if(separ=3);then;do;
? (3) (((K,L),E),M)
  frml sepkmem bkm=1/pkm * blm*plm * (bkk*pkk+bkl*pk1)
    /(bkl*plk+bll*p11);
  frml sepklel bke=1/pke * bel*ple * (bkk*pkk+bkl*pk1)
    /(bkl*plk+bll*p11);
  frml seplmem bem=1/pem * blm*plm * (bke*pek+bee*pee+bel*pel)
    /(bkl*plk+bel*ple+bll*p11);
  eqsub seplmem sepklel;
enddo;

frml s1 blk=bkl;
frml s2 bek=bke;
frml s3 bel=ble;
frml s4 bmk=bkm;
frml s5 bml=blm;
frml s6 bme=bem;

frml ss1 pkk=1;
frml ss2 pkl=sqrt(pl/pk);
frml ss3 pke=sqrt(pe/pk);
```



```
frml ss4  pkm=sqrt(pm/pk);

frml ss5  plk=sqrt(pk/pl);
frml ss6  pll=1;
frml ss7  ple=sqrt(pe/pl);
frml ss8  plm=sqrt(pm/pl);

frml ss9  pek=sqrt(pk/pe);
frml ss10 pel=sqrt(pl/pe);
frml ss11 pee=1;
frml ss12 pem=sqrt(pm/pe);

frml ss13 pmk=sqrt(pk/pm);
frml ss14 pml=sqrt(pl/pm);
frml ss15 pme=sqrt(pe/pm);
frml ss16 pmm=1;

frml sss1 pk=pk_96;
frml sss2 pl=pl_96;
frml sss3 pe=pe_96;
frml sss4 pm=pm_96;

eqsub(print) sepklel s1-s6 ss1-ss16 sss1-sss4;
eqsub(print) sepkmem s1-s6 ss1-ss16 sss1-sss4;
eqsub(print) seplmem s1-s6 ss1-ss16 sss1-sss4;

endproc;

? -----LONG RUN ELASTICITIES-----
proc ela e96 ekk ell eee emm ekl eke
      ekm elk ele elm eek eel eem emk eml eme;

frml elakk ekk=-0.5*(1-(bkk+zk*tid_96+zzk*tid_96*tid_96)*qy/kstar);
frml elall ell=-0.5*(1-(bll+zl*tid_96+zzl*tid_96*tid_96)*qy/lstar);
frml elae eee=-0.5*(1-(bee+ze*tid_96+zze*tid_96*tid_96)*qy/estar);
frml elamm emm=-0.5*(1-(bmm+zm*tid_96+zzm*tid_96*tid_96)*qy/mstar);
frml elakl ekl=0.5*(qy/kstar)*bkl*sqrt(pl/pk);
frml elake eke=0.5*(qy/kstar)*bke*sqrt(pe/pk);
frml elakm ekm=0.5*(qy/kstar)*bkm*sqrt(pm/pk);
frml elalk elk=0.5*(qy/lstar)*bkl*sqrt(pk/pl);
frml elale ele=0.5*(qy/lstar)*ble*sqrt(pe/pl);
frml elalm elm=0.5*(qy/lstar)*blm*sqrt(pm/pl);
frml elaek eek=0.5*(qy/estar)*bke*sqrt(pk/pe);
frml elael eel=0.5*(qy/estar)*ble*sqrt(pl/pe);
frml elaem eem=0.5*(qy/estar)*bem*sqrt(pm/pe);
frml elamk emk=0.5*(qy/mstar)*bkm*sqrt(pk/pm);
frml elaml eml=0.5*(qy/mstar)*blm*sqrt(pl/pm);
frml elame eme=0.5*(qy/mstar)*bem*sqrt(pe/pm);

dot k l e m;
  dot k l e m;
    genr ela..;
  enddot;
enddot;

load(nrow=4,ncol=4,type=general)e96;0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
set e96(1,1)=ekk(1996:1);
set e96(1,2)=ekl(1996:1);
set e96(1,3)=eke(1996:1);
set e96(1,4)=ekm(1996:1);
set e96(2,1)=elk(1996:1);
set e96(2,2)=ell(1996:1);
set e96(2,3)=ele(1996:1);
set e96(2,4)=elm(1996:1);
set e96(3,1)=eek(1996:1);
set e96(3,2)=eel(1996:1);
set e96(3,3)=eee(1996:1);
set e96(3,4)=eem(1996:1);
set e96(4,1)=emk(1996:1);
set e96(4,2)=eml(1996:1);
set e96(4,3)=eme(1996:1);
set e96(4,4)=emm(1996:1);
endproc;
```